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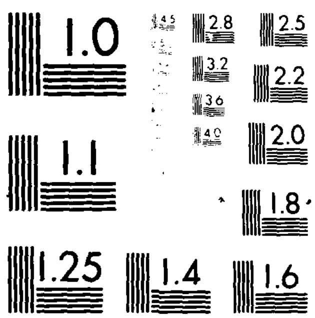

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DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

NASTRAN THEORY AND APPLICATION COURSE SUPPLEMENT

by

Gordon C. Everstine & Myles M. Hurwitz

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Computation, Mathematics, & Logistics Department
Departmental Report

February 1981 DTNSRDC/CMLD-81-05

NASTRAN THEORY AND APPLICATION COURSE SUPPLEMENT

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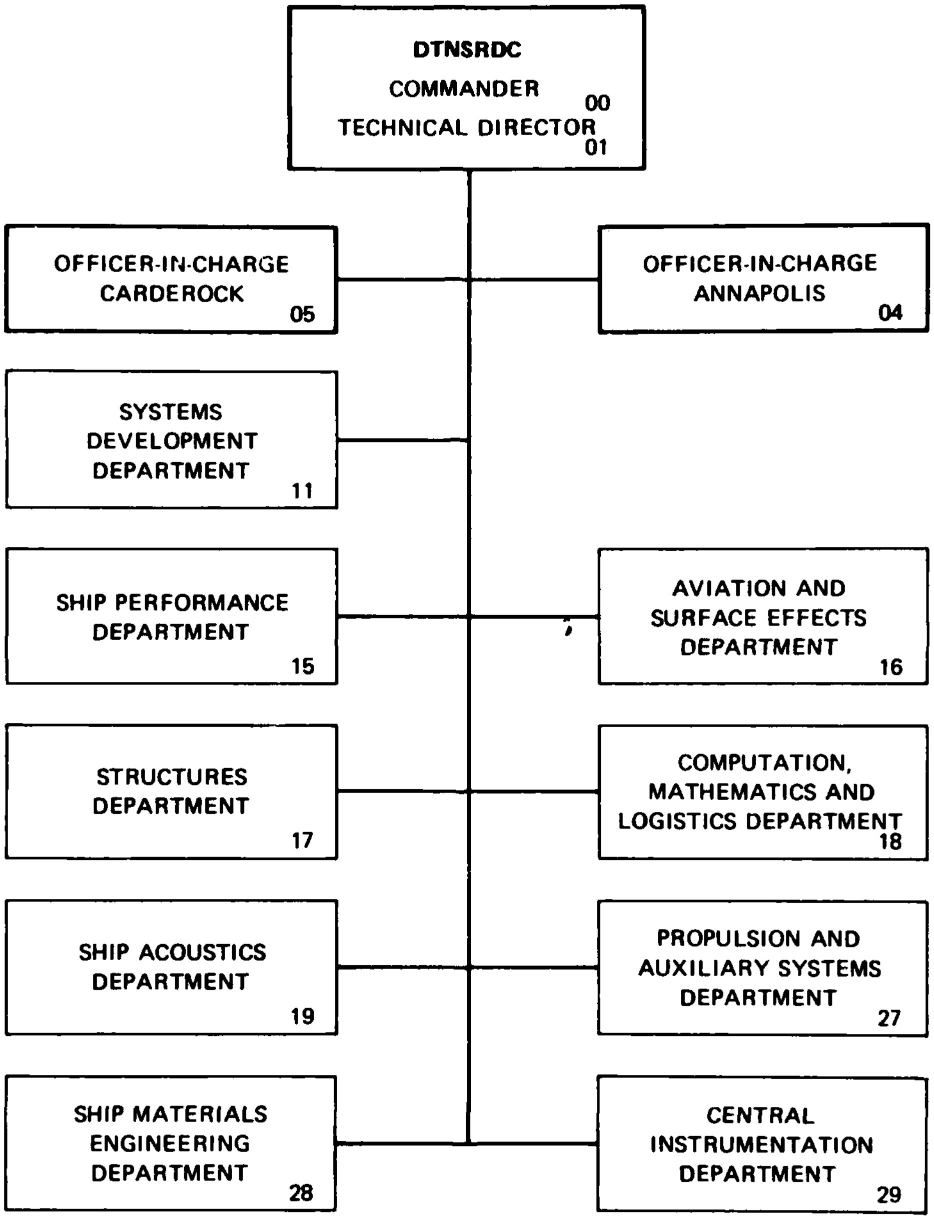

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This is a collection of notes used primarily in training courses on the NASTRAN structural analysis computer program.

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NASTRAN Course Schedule
(90 - Minute Sessions)

	Monday	Tuesday	Wednesday	Thursday	Friday
I	NASTRAN Overview	Static Stress Analysis III; Workshop	Dynamics I - Normal Modes Analysis	Dynamics III - Transient Analysis	Basic Plotting
II	Introduction to Matrix Methods and the Finite Element Method	Workshop	Workshop	Workshop	Large Problem Considerations: G.P. Seque[illegible]ing Time & Cor[illegible] Est.
III	Static Stress Analysis I	Workshop	Dynamics II - Frequency Response	Problem Control and Miscellaneous Topics	Nonstructural Problems
IV	Static Stress Analysis II	Element Library	Workshop	Structural Symmetry	Workshop (Optional)

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14. Proceedings of the Navy-NASTRAN Colloquia, held at Naval Ship Research and Development Center, Bethesda, Maryland:
   (1) 12-13 January 1970 (AD764298 at DDC)
   (2) 7-9 December 1970 (AD764507)
   (3) 22-23 March 1972 (AD764299)
   (4) 27 March 1973 (AD764508)
   (5) 10 September 1974 (ADA004604)

15. NASTRAN: Users' Experiences, Colloquia held at NASA-Langley Research Center, Hampton, Virginia:
   (1) NASA TM X-2378, 13-15 September 1971
   (2) NASA TM X-2637, 11-12 September 1972
   (3) NASA TM X-2893, 11-12 September 1973
   (4) NASA TM X-3278, 9-11 September 1975
   (5) NASA TM X-3428, 5-6 October 1976
   (6) NASA CP-2018, 4-6 October 1977
   (7) NASA CP-2062, October 1978
   (8) NASA CP-2131 , October 1979
   (9) NASA CP-2151, October 1980

Newsletter:

16. NASTRAN Newsletter, published by the Computer Software Management and Information Center (COSMIC), Suite 112, Barrow Hall, Athens, Georgia 30602.

An Introduction to the Finite Element Method

The finite element method views a structure as an assemblage of structural elements interconnected at a finite number of node points. Consider the plane elasticity problem in Figure 1.

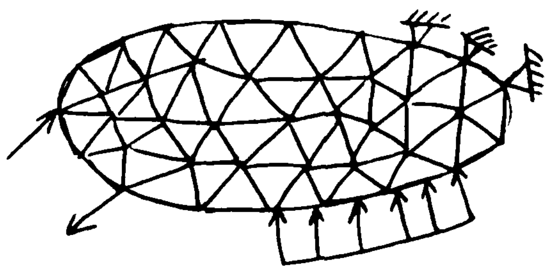


Figure 1

We wish to determine both the deformation and stress fields for the region loaded and constrained as shown.

The essence of the finite element method is as follows:

(1) subdivide the continuum into a number of small simple u elements, as shown in Figure 1

(2) assume a form of the displacement function for each element type (e.g., linear, quadratic, etc.)

(3) derive the stiffness relationship for each element (i.e., find K^e in $K^e u = f$)

(4) assemble the element stiffness matrices K^e into the global stiffness matrix K (this is easy since stiffness can be added algebraically)

(5) solve global matrix equation $Ku = F$ for the displacement vector u (i.e., the components of u are the displacements at the grid points)

(6) knowing the displacements everywhere, compute stresses.

The above will be illustrated by a simple example. We wish to derive a triangular membrane element which is suitable for plane elasticity problems (such as in Figure 1).

Consider the element and coordinate system shown in Figure 2.

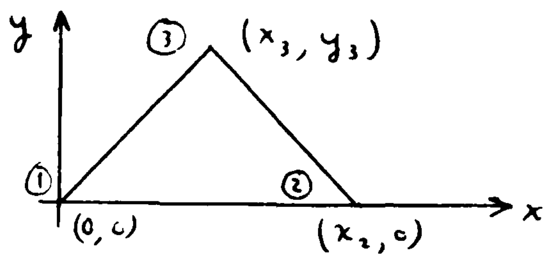


Figure 2

The vertices of the triangle, called grid points or nodes, can displace in either the x- or y-direction. Thus, the element shown has a total of six degrees of freedom (two per node).

We assume displacement functions of the form

$$
\begin{aligned}
u &= a_1 + a_2 x + a_3 y \\
v &= a_4 + a_5 x + a_6 y
\end{aligned} \tag{1}
$$

where u and v are the displacement components in the x and y directions, respectively. Define the vectors $\underline{a}$ and $\underline{u}$ by

$$
\underline{a} = \begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ a_6 \end{Bmatrix} \qquad \underline{u} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \tag{2}
$$

where u_i and v_i are the u and v components at point i .

Writing equation (1) at each node yields

$$\underline{u} = \underline{\gamma}\,\underline{a} \tag{3}$$

where

$$\underline{\gamma} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & x_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_2 & 0 \\ 1 & x_3 & y_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_3 & y_3 \end{bmatrix} \tag{4}$$

The inverse of $\underline{\gamma}$ exists and is given by

$$\underline{\alpha} = \underline{\gamma}^{-1} = \frac{1}{x_2 y_3} \cdot \begin{bmatrix} x_2 y_3 & 0 & 0 & 0 & 0 & 0 \\ -y_3 & 0 & y_3 & 0 & 0 & 0 \\ x_3 - x_2 & 0 & -x_3 & 0 & x_2 & 0 \\ 0 & x_2 y_3 & 0 & 0 & 0 & 0 \\ 0 & -y_3 & 0 & y_3 & 0 & 0 \\ 0 & x_3 - x_2 & 0 & -x_3 & 0 & x_2 \end{bmatrix} \tag{5}$$

The strain components of interest will be grouped into a strain vector $\underline{\epsilon}$:

$$\underline{\epsilon} = \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{Bmatrix} \tag{6}$$

For this example,

$$\underline{\epsilon} = \begin{Bmatrix} a_2 \\ a_6 \\ a_3 + a_5 \end{Bmatrix} \tag{7}$$

Relating the strain vector $\underline{\epsilon}$ to the vector $\underline{a}$ defines $\underline{B}$:

$$\underline{\epsilon} = \underline{B}\,\underline{a} \tag{8}$$

where, in this example,

$$\underline{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{9}$$

Thus,

$$\underline{\epsilon} = \underline{B}\,\underline{a} = \underline{B}\,\underline{\alpha}\,\underline{u} \tag{10}$$

Corresponding to the strain vector is the stress vector

$$\underline{\sigma} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} \tag{11}$$

For linear, elastic materials, the stress and strain vectors are related by Hooke's Law:

$$\underline{\sigma} = \underline{D}\,\underline{\epsilon} \tag{12}$$

where $\underline{D}$ is a matrix of material constants. For example, for plane stress isotropy, $\underline{D}$ is given by

$$\underline{D} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{13}$$

where E and ν are the Young's modulus and Poisson's ratio for the material.

Thus, from Equations (10) and (12),

$$\underline{\sigma} = \underline{D}\,\underline{B}\,\underline{\alpha}\,\underline{u} \tag{14}$$

To summarize, Equation (14) gives a formula for the stress components for an element given the displacements $\underline{u}$ of the node points. The matrices $\underline{D}$, $\underline{B}$, and $\underline{\alpha}$ depend only on material properties and geometry.

The final step for the element is to compute the element stiffness matrix $\underline{k}$. The general result will be

$$\underline{k} = \int_V \underline{\alpha}^T \underline{B}^T \underline{D}\,\underline{B}\,\underline{\alpha}\, dV \tag{15}$$

where the integration is performed over the volume of the element.

To derive (15), let $\underline{P}$ be the vector of forces at the nodes. Applying an arbitrary virtual displacement $\delta \underline{u}$ at the nodes, (10) yields

$$\delta \underline{\epsilon} = \underline{B}\,\underline{\alpha}\, \delta \underline{u} \tag{16}$$

By the principle of virtual work, the work done by $\underline{P}$ at the nodes must balance the internal dissipation of energy; thus,

$$\delta \underline{u}^T \underline{P} = \int_V \delta \underline{\epsilon}^T \underline{\sigma}\, dV \tag{17}$$

Substituting (16) and (14) into (17) yields

$$\begin{aligned} \delta \underline{u}^T \underline{P} &= \int_V \delta \underline{u}^T \underline{\alpha}^T \underline{B}^T \underline{D}\,\underline{B}\,\underline{\alpha}\, \underline{u}\, dV \\ &= \delta \underline{u}^T \left(\int_V \underline{\alpha}^T \underline{B}^T \underline{D}\,\underline{B}\,\underline{\alpha}\, dV \right) \underline{u} \end{aligned} \tag{18}$$

For (18) to hold for an arbitrary variation of displacement $\delta \underline{u}$,

$$\underline{P} = \left(\int \underline{\alpha}^T \underline{B}^T \underline{D}\,\underline{B}\,\underline{\alpha}\, dV \right) u \tag{19}$$

By definition, the parenthetical expression in (19) represents the stiffness matrix for the element; hence, (15) is proved.

For the triangular membrane element, all matrices in the integrand of (15) are constant, so that $\underline{k}$ becomes

$$\underline{k} = \frac{1}{2} x_2\, y_3\, t \left(\underline{\alpha}^T \underline{B}^T \underline{D}\,\underline{B}\,\underline{\alpha}\right) \tag{20}$$

where t is the membrane thickness. This matrix is shown on page 3-7.

Note that equations (8), (10), (12), (14), and (15) are general, so that they apply to other finite elements as well as to the triangular membrane.

For a problem with many elements, the stiffness matrix $\underline{k}$ for each element can be computed and assembled into a global stiffness matrix $\underline{K}$ for the entire structure. Then, one must solve the system

$$\underline{K}\,\underline{u} = \underline{F} \tag{21}$$

where $\underline{u}$ is now the vector of nodal displacements for the entire structure, and $\underline{F}$ is the vector of loads applied at the nodes. Equation (21) is basic to the finite element method.

It is apparent that (21) is merely a generalization of the familiar expression for a one-dimensional spring:

$$kx = f \tag{22}$$

Thus, it is not too surprising that, for time-dependent problems, the analogy still holds and one obtains the matrix equation

$$\underline{M}\,\ddot{\underline{u}} + \underline{B}\,\dot{\underline{u}} + \underline{K}\,\underline{u} = \underline{F}(t) \tag{23}$$

Here, $\underline{M}$ and $\underline{B}$ are mass and damping matrices, respectively, and the dots over u indicate differentiation with respect to the independent variable t (time).

In essence, the approach outlined above approximates the displacement solution by a set of piecewise polynomials. It can thus be shown to be a variant of the Rayleigh-Ritz method.

Stiffness Matrix for Plane Stress Triangular Membrane Finite Element

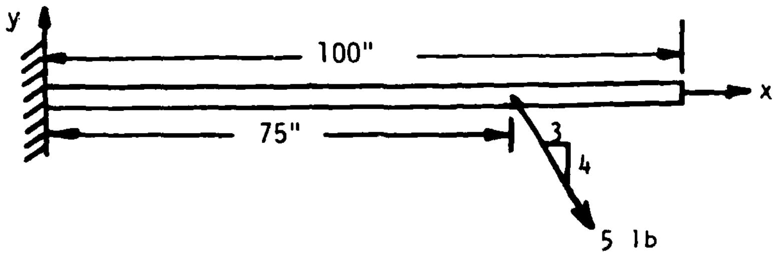
$$\frac{2(1-\nu^2)\, x_2\, y_3}{E\,t}\; \underline{k} =$$

$y_3^2 + \left(\frac{1-\nu}{2}\right)(x_3-x_2)^2$	$-\left(\frac{1+\nu}{2}\right) y_3(x_3-x_2)$	$-y_3^2 - \left(\frac{1-\nu}{2}\right)x_3(x_3-x_2)$	$\left(\frac{\nu+1}{2}\right) x_3y_3 - \left(\frac{1-\nu}{2}\right) x_2y_3$	$\left(\frac{1-\nu}{2}\right) x_2(x_3-x_2)$	$-\nu x_2y_3$
	$\left(\frac{1-\nu}{2}\right)y_3^2 + (x_3-x_2)^2$	$-\nu x_2y_3 + \left(\frac{\nu+1}{2}\right) x_3y_3$	$-\left(\frac{1-\nu}{2}\right) y_3^2 - x_3(x_3-x_2)$	$-\left(\frac{1-\nu}{2}\right)x_2y_3$	$x_2(x_3-x_2)$
		$\left(\frac{1-\nu}{2}\right)x_3^2 + y_3^2$	$-\left(\frac{1+\nu}{2}\right) x_3y_3$	$-\left(\frac{1-\nu}{2}\right)x_2x_3$	νx_2y_3
	SYMMETRIC		$x_3^2 + \left(\frac{1-\nu}{2}\right) y_3^2$	$\left(\frac{1-\nu}{2}\right) x_2y_3$	$-x_2x_3$
				$\left(\frac{1-\nu}{2}\right) x_2^2$	0
					x_2^2

DEMONSTRATION PROBLEMS

See "NASTRAN Sample Problem Computer Output" by G.C. Everstine and M.M. Hurwitz (DTNSRDC/CMLD-81-04) for problem solutions.

1. Cantilever Beam with Point Load

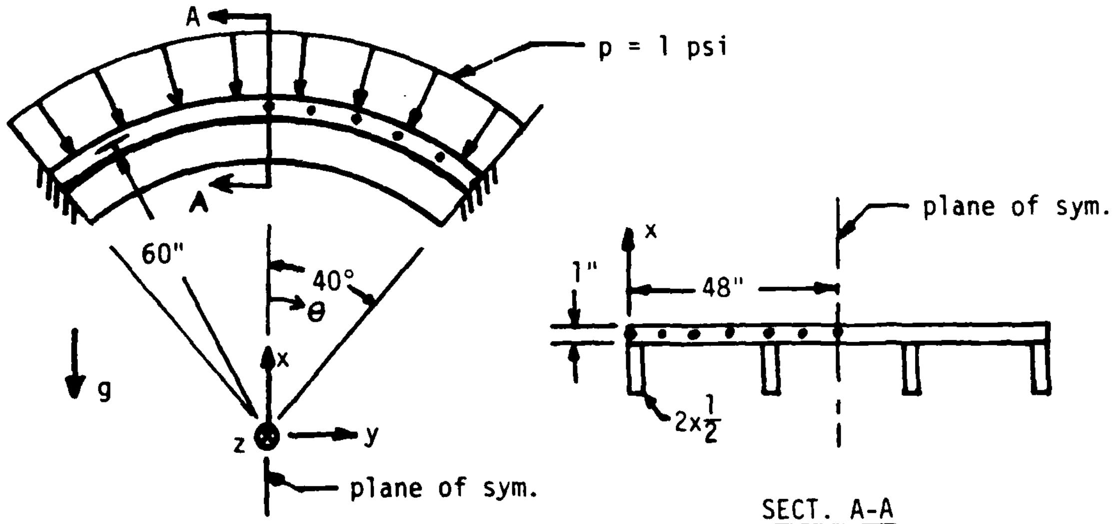


5/8" diameter steel beam

For steel, $E = 30 \times 10^6$ psi, $\nu = 0.3$, $\rho = 7.324 \times 10^{-4}$ lb-sec^2/in^4

Find displacements, stresses, and reactions.

1A. Arch Under Static Pressure



SECT. A-A

Material: Steel

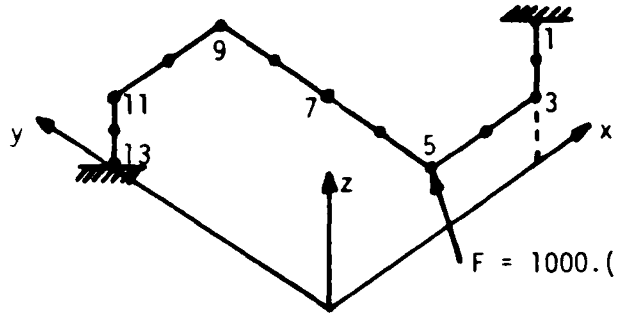
Subcase 1: Pressure Load
Subcase 2: Gravity Load
Subcase 3: Solve a ring-stiffened cylinder problem by changing B.C. at $\theta = \pm 40°$ to planes of symmetry.

1B. Ring-Stiffened Cylinder with Pressure Load

(Modeled using conical shell elements)

Find displacements for same cylinder as in problem 1A, Subcase 3.

1C. Symmetry Example



At Point 7,

Sym. B.C.: $u_x = u_y = u_z = 0$

Anti-Sym.: $R_x = R_y = R_z = 0$

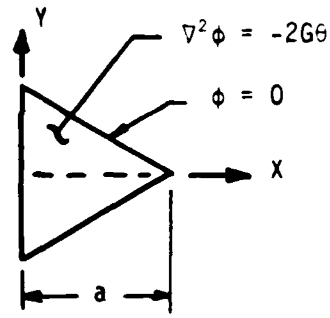
Find displacements by modeling only one-half of structure.

1D. Linear Steady-State Heat Conduction

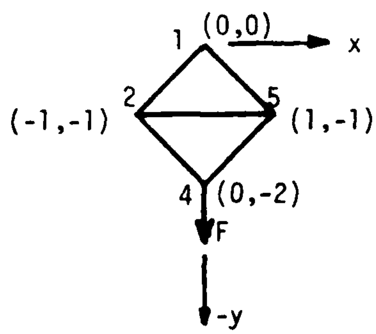
For the arch structure of problem 1A, find the steady-state temperature distribution due to a uniform applied heat flux at $\theta = \pm 40^\circ$ of 0.0025 BTU/sec-in^2, with the edges at Z = 0 and Z = 96 immersed in an ice bath (32°F). The thermal conductivity of steel is 7.175×10^{-4} BTU/sec-in-°F.

1E. 2-D Poisson Equation (Torsion of Triangular Prism)

Compute the maximum shear stress in a twisted bar whose cross-section is an equilateral triangle of altitude a = 0.09 m. The bar has a shear modulus G = 80 GPa and is subjected to an angle of twist per unit length θ = 0.04 rad/m. (The exact solution for the maximum shear stress is $G\theta a/2$.)



2. Static Analysis with Inertia Relief



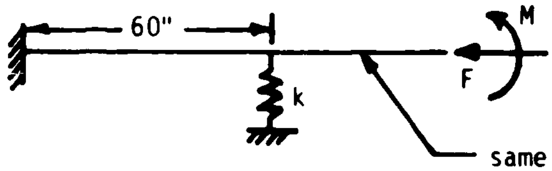
F = 1000., E = 10^7, ν = 0.3

$m_2 = m_5$ = 250., m_1 = 500., A=1.

3. Natural Frequencies and Modes

Find natural frequencies and mode shapes for beam of problem 1. (INV, GIV, FEER, and eigenvalue APPEND)

4. <u>Differential Stiffness</u>



k = 2 lb/in
F = 25 lb
M = 25 lb-in

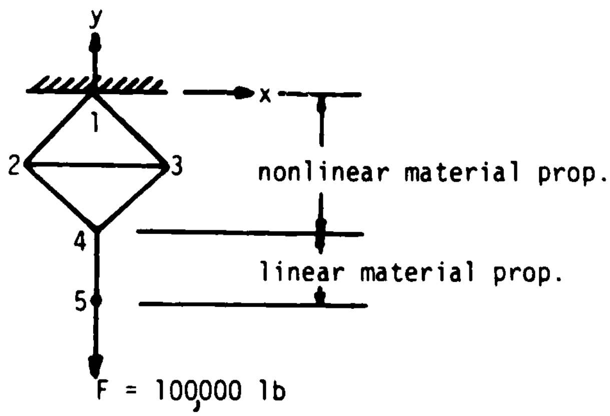
Find transverse deflection.

5. <u>Buckling</u>

For the beam of problem 4, at what value of F and M (assuming F and M are numerically equal) will buckling occur?

6. <u>Piecewise Linear Analysis</u>

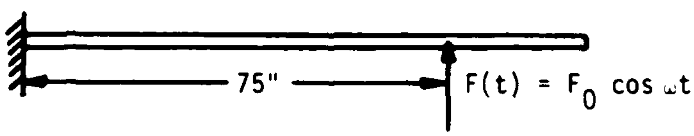
Find the displacements and stresses for the following frame:



7. <u>Complex Eigenvalue Analysis (Direct Method)</u>

Find the damped natural frequencies for the beam of problem 1 with a dashpot (c = 0.25 lb-sec/in) connected in the y-direction between point 13 (at x = 60) and ground.

8. <u>Frequency Response (Direct Method)</u>

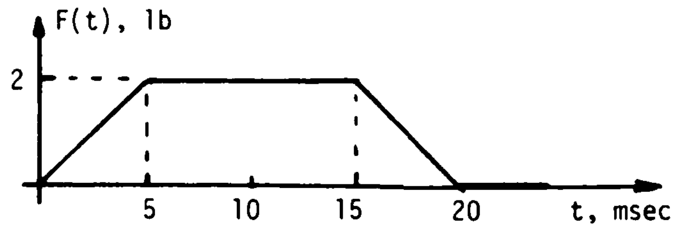


$F_0 = 1$ lb , $\omega = 2\pi f$

Find steady-state displacement response at f = 3 Hz and 7 Hz for beam of problem 1.

9. Transient Response (Direct Method)

Find time-dependent response for beam of problem #8 with F(t) given below:



Problem 9A restarts from t=100 to illustrate TRD CONTINUE.

10. Complex Eigenvalues (Modal Method)

Same as problem #7, except use modal approach.

11.& 11A. Frequency Response (Modal Method)

Same as problem #8, except use modal approach. (INV and GIV)

12. Transient Response (Modal Metnod)

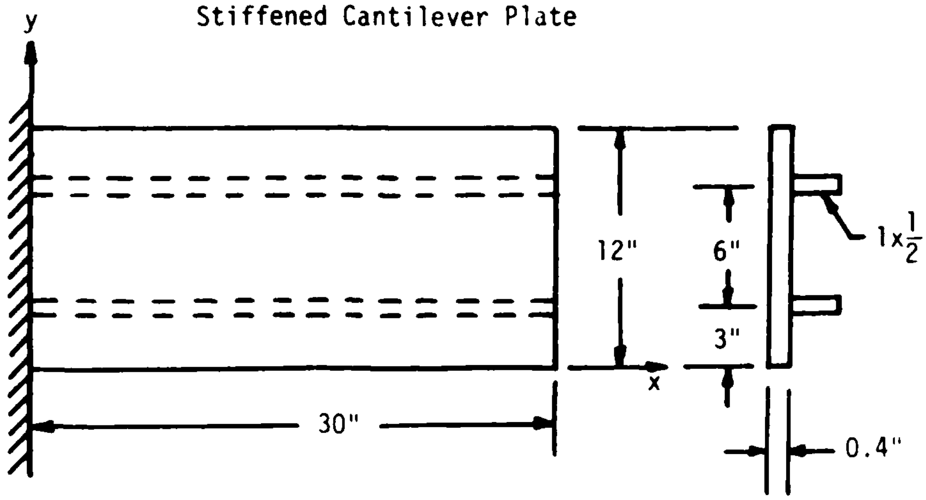
Same as problem #9, except use modal approach.

13. Normal Modes with Differential Stiffness

For the beam of problem #1, find the flexural natural frequencies for the beam spinning about the y axis at 1.5 Hz.

WORKSHOP PROBLEMS

Stiffened Cantilever Plate



material: steel ($E = 30\text{x}10^6$ psi, $\nu = 0.3$, $\rho = 7.324\text{x}10^{-4}$ lb-sec^2/in^4)

This structure will be used for all types of analysis.

Suggested F.E. mesh: 6x4 mesh of plate elements covering the 30"x12" region with beam stiffeners (Note existence of plane of symmetry.)

1. Static Stress Analysis. Determine stresses and displacements for
   a. uniform unit pressure load on plate in -z direction
   b. gravity load in -z direction
   c. sum of (a) and (b)

1a. Plotting.
   a. Plot undeformed structure.
   b. Plot static deformation for pressure load of problem #1.
   c. Make matrix topology plot of constrained stiffness matrix (KLL) for problem #1.

1b. DMAP. For problem #1, starting with the constrained stiffness matrix (KLL) and load vectors (PL), write a DMAP ALTER to R.F. 1 to:
   a. compute displacement vectors (to be called UTEST)
   b. compute the residual (RESID = PL - KLL x UTEST)
   c. compute error ratio

$$\text{ERR1} = \frac{\text{RESID} \cdot \text{UTEST}}{\text{PL} \cdot \text{UTEST}}$$

d. print UTEST, RESID, and ERR1 and check with NASTRAN's values.
e. do problem 1c both before and after grid point resequencing to reduce matrix wavefront.

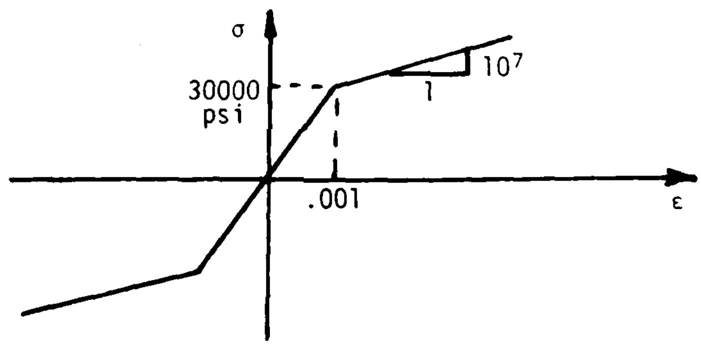
2. Inertia Relief. The plate is free-free (for this problem only) rather than cantilevered. Determine stresses resulting from a 100 lb. normal point force at the center of plate.

3. Normal Modes. Find the lowest 3 natural frequencies and modes.

4. Differential Stiffness. The load is a uniform downward (-z) pressure of 2 psi plus a 20,000 lb uniform line load at the free end in the -x direction. Determine the displacements.

5. Buckling. Find the factor by which the load of problem #4 should be scaled to induce buckling.

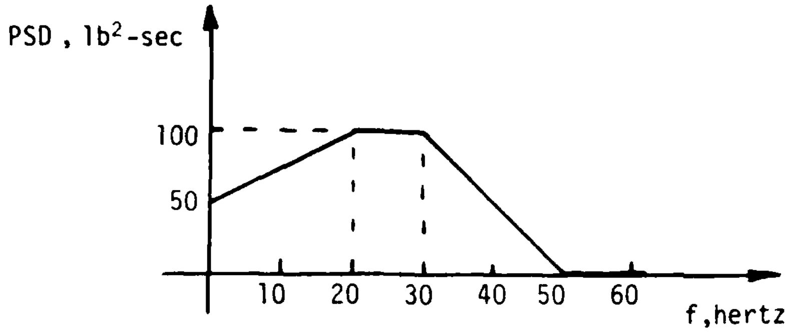
6. Piecewise Linear Analysis. Determine the stresses and displacements for a uniform downward 10 psi pressure load on the plate. Assume that steel has the symmetric stress-strain relation:



7. Complex Eigenvalues. Add uniform viscous dampers in the z-direction which are attached between the plate and ground along y=3 and y=9. The total damping along each line is 2.4 lb-sec/in. Find the damped frequencies and modes.

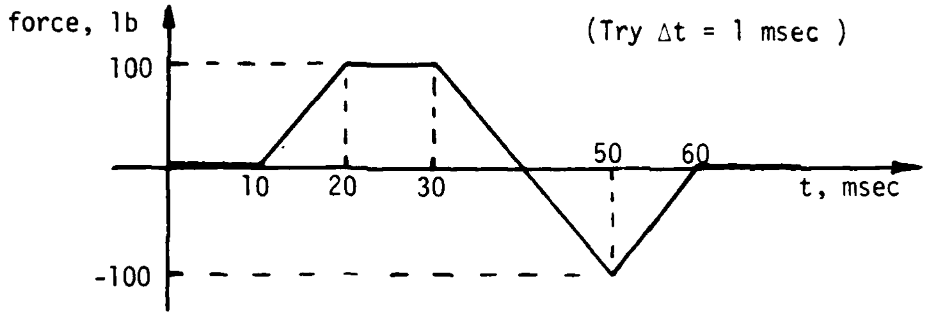
8. Frequency Response. Compute frequency response at the free end to sinusoidal load at the free end of each stiffener: $F_z = 10 \cos 2\pi ft$, where f = 30, 32, 34, ..., 44.

8a. Random Response. Determine the power spectral density (PSD) function and rms value of the z-displacement at the center of the free end (x=30, y=6) for the input normal force PSD (relative to the loading of problem #8) at the free end of each stiffener given on the following page.



8b. Plotting. Make XY printer plots vs. frequency of the responses computed in problems #8 and 8a.

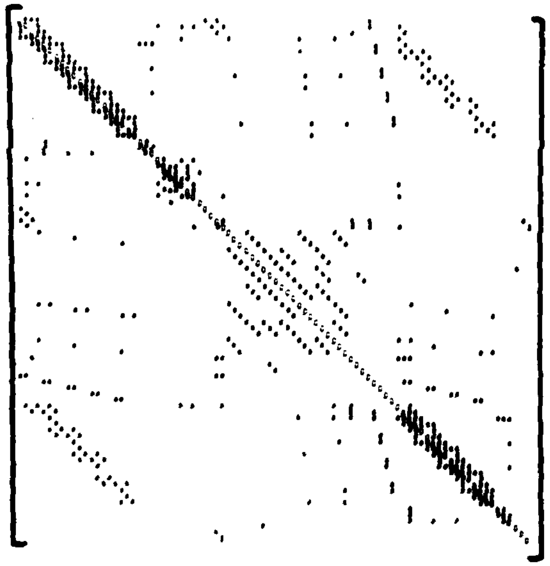
9. Transient Response. Determine the first 75 msec of response at the free end due to the following normal load at the end of each stiffener:



9a. Plotting. Make XY plot (both on paper and on plotter) for transient z-displacement response at center of free end.

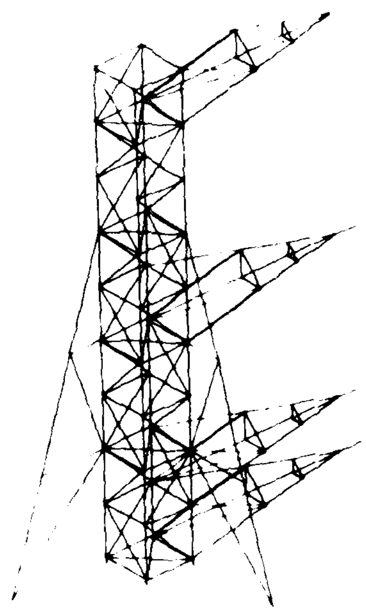
BANDIT EXAMPLE

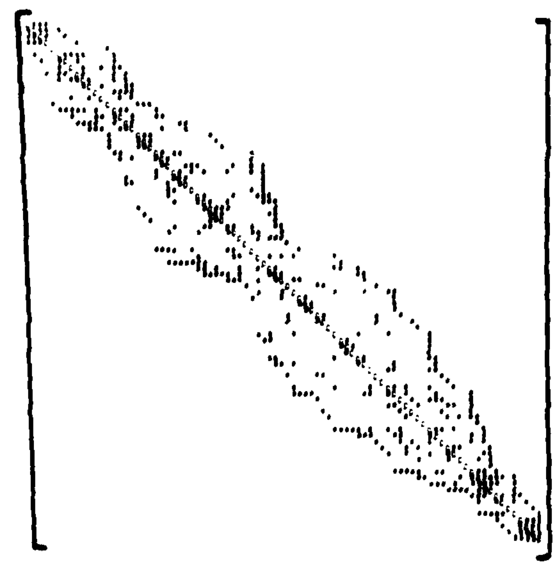
LOCATION OF NON-ZERO TERMS IN STIFFNESS MATRIX BEFORE AND AFTER GRID POINT RESEQUENCING BY BANDIT



BEFORE

(MATRIX BANDWIDTH = 63)





AFTER

(MATRIX BANDWIDTH = 17)

TIME AND CORE ESTIMATION

The general approach to estimating NASTRAN CPU run times is to identify those functional modules which will be most time-consuming, estimate the CPU time for those modules, and then assume that the total CPU run time equals about 140% of the sum of the estimated times.

Every rigid format generally requires stiffness matrix and mass matrix generation and at least one matrix decomposition. The methods for computing these times will be given first. Other details for each rigid format will then be given.

Core requirements are usually thought of as the core required to have a "no spill" condition for decomposition. These requirements will be discussed in the Matrix Decomposition section.

Matrix Generation

The functional modules for stiffness and mass matrix generation are SMA1 and SMA2 for Level 15 (and below) and EMG for Level 16 and above. The CPU times, for various machines, for the generation of one element stiffness matrix, for various elements, are given in Table 7-1 for NASTRAN Levels 15, 16, and 17. The time required to compute an element lumped mass matrix is negligible, while the time to compute an element consistent mass matrix is approximately the same as the time required for the element stiffness matrix.

Matrix Decomposition

The time required to decompose a real, symmetric matrix with no spill is

$$T = \frac{1}{2} M(nb^2 - \frac{2}{3} b^3) \qquad \text{(Level 15 and below)} \qquad (7\text{-}1)$$

$$T = \frac{1}{2} MnC_{rms}^2 \qquad \text{(Level 16 and above)} \qquad (7\text{-}2)$$

where

T = CPU time

n = order of the matrix

b = matrix semi-bandwidth

C_{rms} = matrix rms wavefront

M = machine time constant (see Table 7-3)

Table 7-1.

ELEMENT MATRIX GENERATION TIMES FOR NASTRAN

Element	No. of Gauss Pts.	CDC 6400 Time (Sec)*	Remarks
BAR		0.27	
BEAM		0.22	MSC/NASTRAN
CONEAX		0.43	per harmonic
HEXA1		2.0	
HEXA2		4.0	
HEXA (8 nodes)		0.80	MSC/NASTRAN
HEXA (20 nodes)		5.60	MSC/NASTRAN
HEX20		6.40	MSC/NASTRAN
IHEX1	2	1.0	
	3	2.5	
	4	5.5	
	2	0.2	heat conduction
IHEX2	2	5.0	
	3	12.5	
	3	4.0	single precision (DTNSRDC)
	4	27.5	
IHEX3	3	31.0	
	4	68.0	
IS2D8	3	2.2	single precision (DTNSRDC)
IS3D8	2	2.4	Sperry/NASTRAN
IS3D20	2	13.4	Sperry/NASTRAN
	3	39.2	Sperry/NASTRAN
PENTA (6 nodes)		0.33	MSC/NASTRAN
PENTA (15 nodes)		1.80	MSC/NASTRAN
QDMEM		0.60	
QUAD1,QUAD2		2.02	
QUAD4		0.30	MSC/NASTRAN
ROD		0.06	
SHEAR		0.30	
TETRA		0.40	
TRAPAX		2.20	
TRAPRG		1.05	
TRIAAX		0.55	
TRIA1,TRIA2		1.52	
TRIA3		0.14	MSC/NASTRAN
TRIARG		0.63	
TRIAX6		1.05	MSC/NASTRAN
TRIM6		0.28	
TRMEM		0.15	
TUBE		0.06	
WEDGE		1.20	

*See Table 7-2 for conversion factors for other computers.

Table 7-2.

CONVERSION FACTORS FOR OTHER COMPUTERS

To Convert the CDC 6400 Times in Table 7-1 to	Multiply by
CDC 6500 and Cyber 73	1.0
CDC 6600 and Cyber 74	0.33
CDC 7600 and Cyber 76	0.05
Cyber 172	0.65
173, 174	0.45
175	0.12
176	0.05
IBM 360/50	2.0
65	0.60
75	
85	0.18
91, 95	0.15
370/155	0.75
158	
165	0.18
168	
195	
Univac 1108	0.33

TABLE 7-3

Timing Parameters
(real arithmetic, μ sec)

	M	I_u	P_s
CDC 6400	15	140	12
6600	5	70	5
7600	0.6		2
UNIVAC 1108	14	50	10
IBM 360/65	20	125	50
75	12	75	30
85	2	40	10
91	0.4	50	2
95	0.32	25	2

The timing equations (7-1) and (7-2) are just the expected dominant terms of the full timing equations, which may be found in the Theoretical Manual. For Equation (7-1), the number of active columns outside the band (in the Level 15 sense) is assumed to be small, as would normally be the case if a bandwidth resequencing program is used.

The core required to perform a real, symmetric decomposition with no spill is

$$N = \frac{1}{2} b(b+5)p + PROG \qquad \text{(Level 15 and below)} \qquad (7\text{-}3)$$

$$N = \frac{1}{2} C_{max}^2 p + PROG \qquad \text{(Level 16 and above)} \qquad (7\text{-}4)$$

where

N = the number of <u>decimal</u> <u>words</u> required

b = matrix semi-bandwidth

p = 1 for CDC, 2 for IBM and UNIVAC

PROG (given in <u>decimal</u> <u>words</u>) = 20000 for CDC, 30000 for IBM, 25000 for UNIVAC

C_{max} = matrix maximum wavefront

The first term on the right-hand sides of Equations (7-3) and (7-4) may be termed the working storage W.

The Level 15 formula, Equation (7-3), assumes a small number of active columns.

The bandwidth, b, and wavefront terms, C_{rms} and C_{max}, required in equations (7-1) through (7-4) may be computed, using BANDIT, as follows: Multiply the required term (b, C_{max}, or C_{rms}) obtained from BANDIT by

$$6 - \frac{(MPC + SPC + PS)}{G}$$

where

SPC = number of degrees of freedom SPC'd

MPC = number of dependent degrees of freedom MPC'd

PS = number of degrees of freedom constrained on GRID cards

G = number of grid points in the problem

(This formula represents the average number of free degrees of freedom per grid point.) If a significant number of degrees of freedom are OMIT'd, then it may be assumed that b=n, C_{max}=n, and $C_{rms}=n/\sqrt{3}$. (n=matrix order)

For real, unsymmetric decomposition with no spill, for both Levels 15 and 16, the time and working storage required are each p times the amount required for real, symmetric decomposition on Level 15, where p=4 for CDC, 2 for IBM and UNIVAC.

For Level 16 complex, symmetric decomposition with no spill, the time is 4 times as long as Level 16 real, symmetric decomposition, and the working storage must be twice that required for Level 16 real, symmetric.

For Level 15 complex decomposition, both symmetric and unsymmetric, and for Level 16 complex, unsymmetric decomposition, the time is 16-20 times longer than Level 15 real, symmetric decomposition. The working storage required is p times the amount required for Level 15 real, symmetric decomposition, where p=8 for CDC, 4 for IBM and UNIVAC.

Forward-Backward Substitution (FBS)

Forward-backward substitution (FBS) is the second (and final) step required in the solution of a set of simultaneous linear equations. (Matrix decomposition is the first step.) Normally, FBS times are relatively small. However, it is not uncommon for FBS times to be significant. The time required for FBS is:

$$T = 2bnrM + \left[\frac{rn}{W}\right] 2nbI_u \qquad \text{(Level 15)} \qquad (7\text{-}5)$$

$$T = 2nC_{avg}\left(rM + \left[\frac{rn}{W}\right] P_s\right) \qquad \text{(Level 16)} \qquad (7\text{-}6)$$

where

r = number of right-hand side vectors

C_{avg} = average wavefront (determined from BANDIT)

W = working storage

Once again, the terms in Equations (7-5) and (7-6) are the dominant ones in the full timing equations. Brackets mean "next larger integer".

Matrix Multiply and Add (MPYAD)

$$A_{mn} B_{np} = D_{mp} \qquad (7\text{-}7)$$

$$T = mn\rho_A \left(\rho M + \left[\frac{np + mp}{W} \right] P_s \right) \text{ (Level 16)} \qquad (7\text{-}8)$$

where ρ_A is the density of the A matrix and the timing constants are given in Table 7-3.

Multipoint Constraints (MPC)

In multipoint constraint elimination, three multiply-add (MPYAD) operations are performed. If we use the notation of Equation (7-7), then the matrix orders of the three multiplications are:

1. $m = d$, $n = d$, $p = i$, $\rho =$ density of stiffness matrix (BANDIT)
2. $m = i$, $n = d$, $p = i$, $\rho =$ density of stiffness matrix (BANDIT)
3. $m = i$, $n = d$, $p = i$, $\rho =$ density of multipoint constraint transformation matrix

where:

d = number of MPC equations
i = (total number of degrees of freedom in the problem) - d

The density of the transformation matrix may be estimated as the ratio of the number of degrees-of-freedom in MPC equations to the total number of degrees-of-freedom in the problem.

Guyan Reduction

OMIT partitioning and Guyan Reduction may be time-consuming processes. One real, symmetric matrix decomposition is required, the order of the matrix being the number of degrees of freedom in the o-set, i.e., degrees of freedom omitted. Also, there will be as many FBS's as there are degrees of freedom in the a-set, i.e., degrees of freedom not MPC'd, SPC'd, or OMIT'd.

For some of the more commonly used rigid formats, additional timing details and core requirements are as follows:

Rigid Format 1

In static analysis, usually, the only major time-consuming operations are stiffness matrix generation, real symmetric decomposition, and multipoint constraints.

Rigid Format 3

Time for one eigenvalue (Functional Module READ) by INV = one decomposition + 8 FBS

Time for Givens method = An^3M, where

A = factor of 5-10 depending on the number of eigenvectors requested, 5 for none, 10 for all

n = matrix order

M = multiply times as previously given

The core required for Givens method is computed as though the matrix were full, i.e., bandwidth = maximum wavefront = n.

Rigid Formats 4 and 5

The differential stiffness matrix time is approximately the same as the linear stiffness matrix time.

Rigid Format 8

Time for one requested frequency (Functional Module FRRD) is approximately the time for a matrix decomposition. In Level 15, the decomposition is always complex, unsymmetric. In Level 16, the decomposition depends on the matrix, and, therefore, may be real or complex, symmetric or unsymmetric. The inclusion of DMIG cards will automatically trigger an unsymmetric decomposition.

Rigid Format 9

Time for transient integration (Functional Module TRD) = one decomposition per time step change + two FBS per time step (with r=1 in Equations (7-5) and (7-6))

Inclusion of DMIG cards will automatically trigger an unsymmetric decomposition.

Rigid Formats 11 and 12

The dominant time in these rigid formats will come from the eigenvalue analysis required to compute the requested mode shapes, not from the frequency response or transient response analyses. Therefore, the time will be approximately the same as in Rigid Format 3.

If the user adds the times for stiffness and mass matrix computation to the times given for these rigid formats, and increases the total time by about 40%, the result should be close to the total NASTRAN CPU time.

BASIC PLOTTING

Types

1. structural -

   undeformed or deformed
   orthographic, perspective, or stereoscopic projection

2. X-Y

3. topological displays of matrices
4. contour

Structural Plotting (Theory)

The model is defined in the basic coordinate system (XYZ). The plotter coordinate system is denoted RST. The XYZ system (fixed with respect to the structure) is oriented with respect to the RST system (fixed with respect to the plotter) in two steps:

1. overlay XYZ on RST in some order

2. rotate XYZ w.r.t. RST by the angles γ, β, α (in that order), which are the angles of rotation about the T, S, and R axes, respectively.

For orthographic plots, NASTRAN then plots the projection in the ST-plane. For perspective plots, NASTRAN also needs to know the location of a vantage point and of a projection plane (plotter surface). Stereoscopic plots consist of two perspective images, each with a slightly different vantage point, which are viewed simultaneously using a stereo viewer.

Deformed plots also require the user to specify the scaling to be applied to the deformation.

Structural Plotting (Practice)

1. All NASTRAN plots are written on a file called PLT1 or PLT2 (usually PLT2). Therefore, using system control cards, user must request computer operator to mount a plot tape of that name. At some installations, plot files can be written to disk.

2. Insert plot control cards into the case control deck immediately before the Begin Bulk card. (See next section.)

3. If necessary, request that the plot tape be processed (i.e., plotted) by the plotter.

Plot Control Cards (Structural Plots)

1. Example 1: undeformed plot

```
PLOTID = JOHN DOE, NSRDC 1844
OUTPUT(PLOT)
PLOTTER SC MODEL 4020
SET 1 = 4, 8, 10 THRU 27, 41
ORTHOGRAPHIC PROJECTION
AXES Z, X, Y
VIEW  -40., 30., 0.
FIND SCALE, ORIGIN 1, SET 1
PLOT SET 1
```

2. Example 2: deformed plot

   Insert the following before FIND card:

```
MAXIMUM DEFORMATION  3.
```

   Replace PLOT card with

```
PLOT STATIC DEFORMATION, SET 1
```

Plot Control Cards (XY Plotting)

1. Example 1:

```
PLOTID = JOHN DOE, NSRDC 1844
OUTPUT(XYPLOT)
PLOTTER = SC 4020
parameter cards (optional)
XYPLOT DISP RESPONSE 2,5/16 (T1)
```

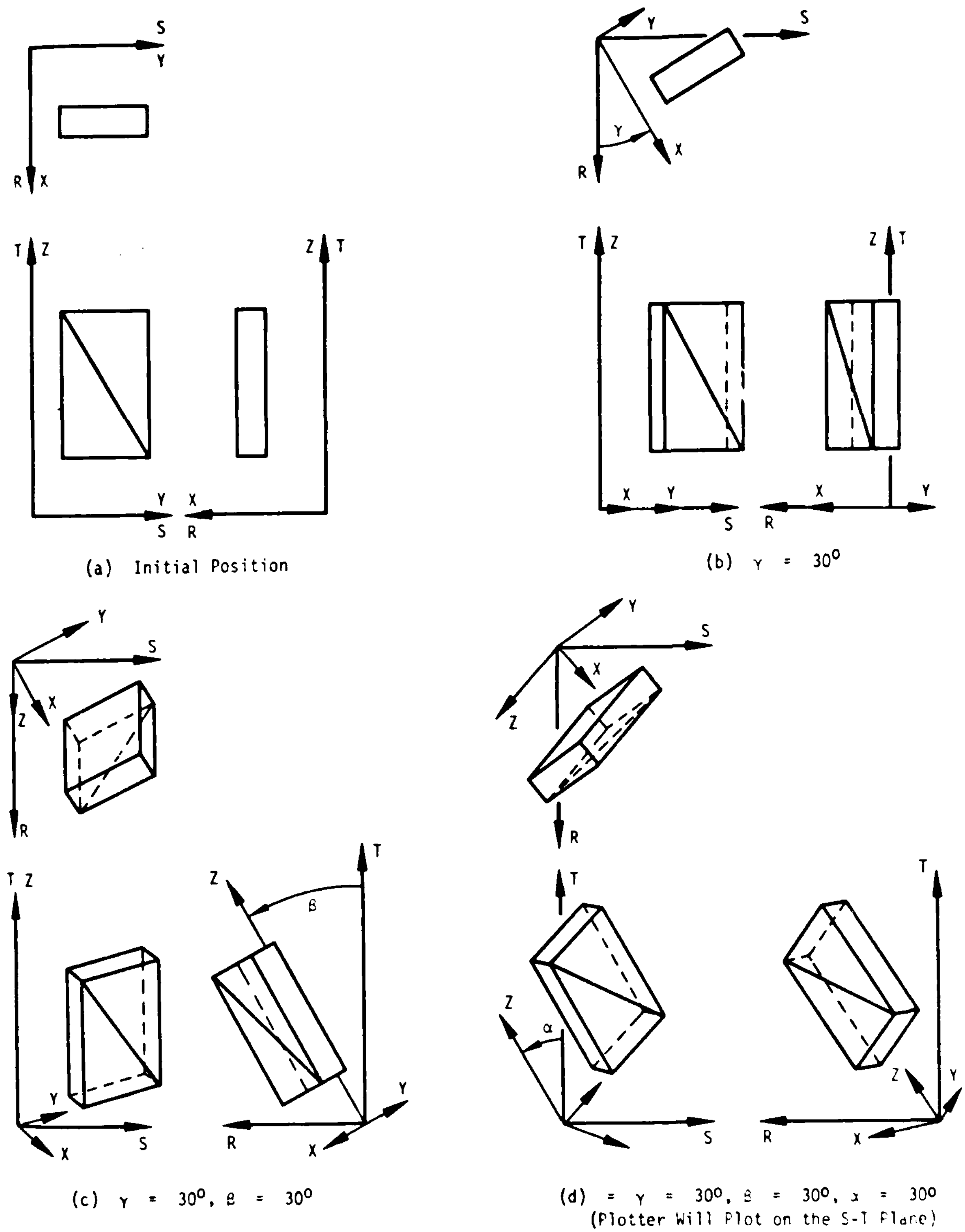
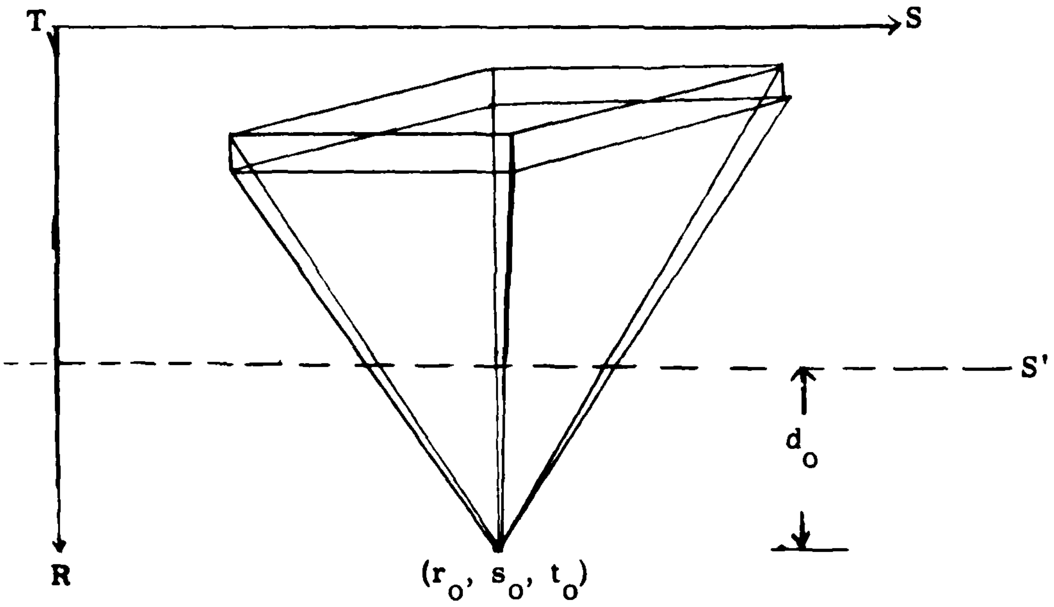


Figure 2. Plotter — model orientation.

13.1-7 (NASTRAN Theoretical Manual)



Perspective Projection

NASTRAN Plotting at DTNSRDC

The CALCOMP plotter, model 936, is used to process NASTRAN plots at DTNSRDC. Since this model is not supported directly by NASTRAN, the "plotter" specified by the user is NASTPLT, the general purpose plotter package. The plotting output written by NASTRAN on file PLT2 is then interpreted by a post-processor called PLTTRN936, which was written by J.M. McKee. The output from PLTTRN936 is written on a tape (assigned by the user) and processed by the CALCOMP 936.

Thus, the PLOTTER card in the plot request packet of the NASTRAN Case Control Deck is

PLØTTER NASTPLT, MØDEL T, 0

The additions to the CDC system control cards are as follows:

1. Prior to the NASTRAN execution card, insert

   REQUEST,PLT2,*PF.

2. After the NASTRAN execution card, insert

```
CATALØG,PLT2,...
REWIND,PLT2.
ATTACH,PLTTRN,PLTTRN936,ID=CAMY,MR=1.
UNLØAD,TAPE3.
REQUEST,TAPE3,HI,S,RING,VSN=XXXXXX.
PLTTRN.
```

3. The MT parameter on the job card must be increased, if necessary, by unity.

In order for PLT2 to be written on disk rather than on tape, PLT2 should be listed after the FILES parameter of the NASTRAN card, which precedes the ID card. Thus, for example,

NASTRAN CØNFIG=6,FILES=(NPTP,ØPTP,PLT2)

A plot request (form 10462/26) must be submitted to ADP Control for each tape.

NASTRAN ELEMENTS

Elements are defined on connection cards (e.g., CBAR, CROD), which list the grid points to which the elements are connected and refer to property cards (e.g., PBAR, PROD), which define geometrical properties (e.g., A, I, J, t, etc.) and in turn refer to material property cards (e.g., MAT1), which define material properties (e.g., E, G, ν, ρ).

The elements contained in level 17 are summarized and categorized in Table 9-1 (p. 9-8) and described (by category) as follows:

One-dimensional elements

CBAR (beam)

- based on simple beam theory
- assumes uniform properties over the length
- assumes shear center coincides with elastic axis
- includes extension, torsion, bending in two planes, shear
- ends may be offset from defining grid points
- any 5 of 6 forces at each end may be set equal to zero using pin flags
- 6 DOF/point

CROD (rod)

- special case of beam with only axial and torsional properties
- no offsets or pin flags
- 2 DOF/point (u_x, R_x)

CONROD (rod)

- same as ROD except properties are included on connection card

CTUBE (tube)

- a rod of circular cross-section, either solid or hollow

CVISC (viscous rod)

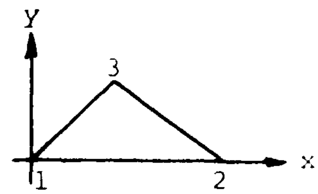
- extensional and torsional viscous damping properties rather than stiffness properties

Two-dimensional elements

CTRMEM (triangular membrane)

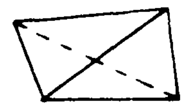
constant strain triangle (CST)

2 DOF/point (u_x, u_y)



CQDMEM (quadrilateral membrane)

two pairs of overlapping TRMEM's



CQDMEM1 (quadrilateral membrane)

linear isoparametric quadrilateral membrane element

2 DOF/point (u_x, u_y)

inefficient since a 4x4 Gauss quadrature is performed instead of 2x2

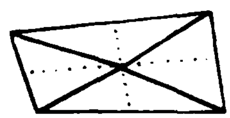
most accurate of the quadrilateral membranes in NASTRAN

CQDMEM2 (quadrilateral membrane)

four non-overlapping TRMEM's

2 DOF/point (u_x, u_y)

more accurate than QDMEM



CTRBSC (basic bending triangle)

3 DOF/point (u_z, R_x, R_y)

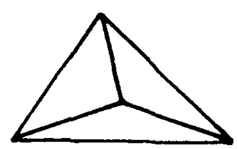
normal displacement u_z varies as (incomplete) cubic in x and y

used as building block for other elements rather than as stand-alone

CTRPLT (triangular bending plate)

three non-overlapping basic bending triangles joined at centroid (the Clough bending triangle)

3 DOF/point (u_z, R_x, R_y)



CQDPLT (quadrilateral bending plate)

two pairs of overlapping basic bending triangles

3 DOF/point (u_z, R_x, R_y)

CTRIAi (membrane and bending triangle)

superposition of TRMEM and TRPLT

5 DOF/point (u_x, u_y, u_z, R_x, R_y)

i = 1: nonhomogeneous panel (e.g., sandwich or honeycomb construction

i = 2: homogeneous panel

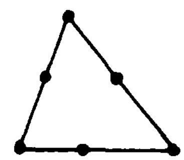
CQUADi (membrane and bending quadrilateral)

superposition of QDMEM and QDPLT

5 DOF/point (u_x, u_y, u_z, R_x, R_y)

i = 1,2: same as for TRIAi

CTRIM6 (triangular membrane)



linear strain triangle (LST)

six nodes (three corner, three mid-side)

thickness can vary bilinearly in x and y

2 DOF/point (u_x, u_y)

most accurate of the NASTRAN membranes

CTRPLT1 (triangular bending plate)

six nodes (three corner, three mid-side)

3 DOF/point (u_z, R_x, R_y)

normal displacement u_z varies as (incomplete) quintic in x and y

thickness can vary bilinearly in x and y

CTRSHL (triangular shallow shell)

a combination of TRIM6 and TRPLT1

shell surface (which need not be flat) is approximated quadratically in x and y

5 DOF/point (u_x, u_y, u_z, R_x, R_y)

CSHEAR (quadrilateral shear panel)

resists the action of tangential forces applied to its edges but not the action of normal forces

usually used in combination with rods or beams

1 DOF/point (in-plane along diagonal)

CTWIST (quadrilateral twist panel)

bending analog of shear panel

equivalent for bending action to a pair of parallel shear panels

1 DOF/point (moment having axis perpendicular to diagonal and in-plane)

Three-dimensional elements

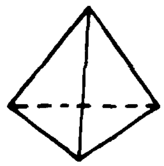
CTETRA (tetrahedron)

constant strain tetrahedron

3-D analog of TRMEM (CST)

4 vertices, 4 triangular faces

3 DOF/point (u_x, u_y, u_z)

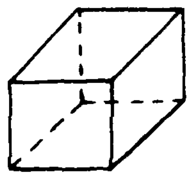


CHEXAi (hexahedron)

8 vertices, 6 quadrilateral faces

superposition of 5 non-overlapping TETRA elements (HEXA1) or 10 overlapping TETRA elements (HEXA2)

3 DOF/point (u_x, u_y, u_z)

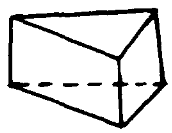


CWEDGE (wedge)

6 vertices, 3 quadrilateral faces and 2 triangular faces

superposition of 3 TETRA's

3 DOF/point (u_x, u_y, u_z)



CIHEXi (isoparametric hexahedron)

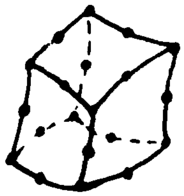
8 vertices, 6 faces

linear (i=1), quadratic (i=2), or cubic (i=3) shape functions

8 (i=1), 20 (i=2), or 32 (i=3) nodes

isotropic materials only

3 DOF/point (u_x, u_y, u_z)



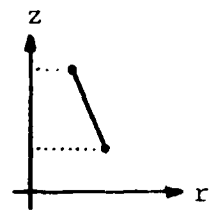
Axisymmetric Elements

CCONEAX (conical shell)

for axisymmetric thin shells

includes membrane, bending, and transverse shear effects

loads may be non-axisymmetric since motions are expanded in Fourier series in aximuthal coordinate



5 DOF/point (normal rotation is excluded)

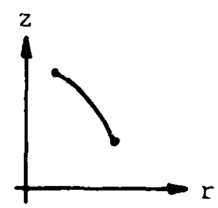
CTORDRG (doubly curved toroidal ring)

for axisymmetric thin shells

axisymmetric loads only

includes membrane and flexural behavior

membrane displacement function is complete cubic



flexural displacement function is complete quintic

5 DOF/point (u_θ is excluded)

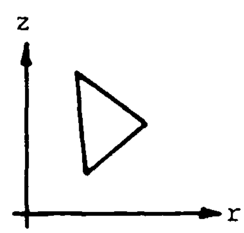
CTRIARG (triangular ring)

solid of revolution element for thick-walled axisymmetric structures

loads must be axisymmetric

linear displacement function

2 DOF/point (u_r, u_z)



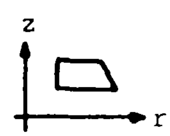
CTRIAAX (triangular ring)

generalization of TRIARG which allows non-axisymmetric deformation by expanding motions in Fourier series in aximuthal coordinate

3 DOF/point (u_r, u_θ, u_z)

CTRAPRG (trapezoidal ring)

similar to TRIARG except trapezoidal shape



CTRAPAX (trapezoidal ring)

similar to TRIAAX except trapezoidal shape

Miscellaneous Structural Elements

CONMi (concentrated mass)

allows input of a 6x6 symmetric mass matrix at a grid point

CDUMi (dummy element)

an element for which the user has written his own FORTRAN subroutines and incorporated them into NASTRAN

Scalar Elements (connect DOF's rather than grid points)

CELASi (scalar spring) 

CMASSi (scalar mass)

CDAMPi (scalar damper) 

Rigid Elements

CRIGD1 (rigid element)

all 6 DOF of dependent grid points are coupled to all 6 DOF of the reference grid point

CRIGD2 (rigid element)

selected DOF of dependent grid points are coupled to all 6 DOF of the reference grid point

CRIGD3 (general rigid element)

selected DOF of dependent grid points are coupled to 6 selected DOF of one or more reference grid points

CRIGDR (rigid rod)

a rod which is rigid in extension/compression

Non-structural Elements

CAXIFi (axisymmetric fluid element)

CFLUIDi (axisymmetric fluid element)

CSLOTi (acoustic cavity slot element)

CHBDY (heat transfer boundary element)

Miscellaneous

CNGRNT (identical elements indicator)

designates secondary elements identical to a primary element to avoid regeneration of the stiffness and mass matrices

TABLE 9-1 - Element Summary

1-D	2-D	3-D	AXI-SYM.
CBAR	CTRMEM	CTETRA	CCONEAX
CROD	CQDMEM	CHEXAi	CTORDRG
CONROD	CQDMEM1	CWEDGE	CTRIARG
CTUBE	CQDMEM2	CIHEXi	CTRIAAX
CVISC	CTRBSC		CTRAPRG
	CTRPLT		CTRAPAX
	CQDPLT		
	CTRIAi		
SCALAR	CQUADi	RIGID	NON-STRUCT.
CELASi	CTRIM6	CRIGDi	CAXIFi
CMASSi	CTRPLT1	CRIGDR	CFLUIDi
CDAMPi	CTRSHL		CSLOTi
	CSHEAR		CHBDY
	CTWIST		
MISC. STRUCT.		MISC.	
CONMi		CNGRNT	
CDUMi			

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THE APPLICATION OF STRUCTURAL SYMMETRY IN FINITE ELEMENT ANALYSIS

Gordon C. Everstine

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COMPUTATION, MATHEMATICS, AND LOGISTICS DEPARTMENT
TECHNICAL MEMORANDUM

JULY 1977 TM-184-77-05

THE APPLICATION OF STRUCTURAL SYMMETRY
IN FINITE ELEMENT ANALYSIS

Summary. A brief review is presented of the fundamental concepts involved in the systematic application of structural symmetry in finite element analysis.

BACKGROUND

Since structural analysts are frequently called upon to analyze structures possessing symmetry, it is essential that the fundamental concepts of symmetry be sufficiently well understood that symmetry can be exploited systematically and with confidence [1,2]. The motivation for wanting to exploit symmetry is clear: when symmetry is present, the engineer need model only a portion of the structure, thereby saving both his time and the computer's time, with the former probably being the more valuable. For example, a structure possessing one plane of symmetry can be analyzed by modeling only one-half of the structure, whether the loads are symmetric or not. Even with nonsymmetric loads, in which case the half-structure would have to be analyzed twice, the analyst still benefits, since a half-structure generally costs much less than half as much to analyze as the complete structure would.

SOME MOTIVATING EXAMPLES

Consider first the following two-dimensional example of a simply-supported beam

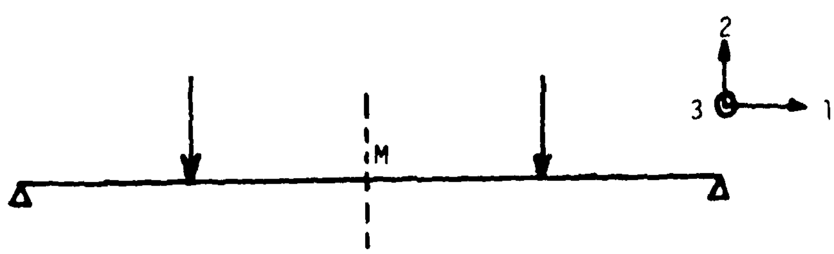

Figure 1

The symmetry present here is probably obvious, so that the analyst who wanted to model only one-half of the beam's span would determine (probably by inspection) that the symmetry boundary conditions to impose at mid-span are

$$u_1 = 0, \quad R_3 = 0 \tag{1}$$

(for the 2-D problem) where u_1 and R_3 are components of the general 3-D displacement and rotation vectors

$$\underline{u} = u_1 \, \underline{i} + u_2 \, \underline{j} + u_3 \, \underline{k}$$
$$\underline{R} = R_1 \, \underline{i} + R_2 \, \underline{j} + R_3 \, \underline{k} \tag{2}$$

(It is assumed here that grid points possess six degrees of freedom (DOF). Generalization to situations with more DOF per node presents no problem.)

Only slightly less obvious than the situation in Figure 1 is the following

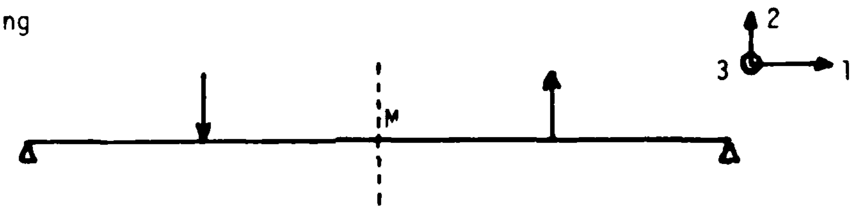

Figure 2

in which the load is now antisymmetric. In this case, the antisymmetry boundary condition to be applied at mid-span is

$$u_2 = 0 \tag{3}$$

a condition which many analysts would probably arrive at by inspection.

Consider now the following two-dimensional beam structure

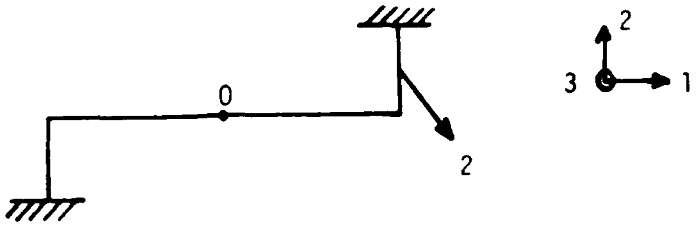

Figure 3

Like the preceding two examples, this problem can also be solved by modeling only half the structure and applying the appropriate symmetry boundary conditions. However, unlike the preceding two examples, the reliance on intuition alone in the application of symmetry would probably fail. Thus, what is needed is a systematic procedure to follow with regard to symmetry. The following sections summarize such a procedure.

TYPES OF SYMMETRY

In structural mechanics, the most commonly encountered types of symmetry are planes of symmetry, axes of symmetry, and centers of symmetry. Each type of symmetry is characterized by some symmetry operation (reflection, rotation, etc.) which can transform the structure into an equivalent configuration [2]. The symmetry operation which characterizes a plane of symmetry is reflection in a plane, as, for example, in Figure 1. An axis of symmetry, exemplified by the structure in Figure 4,

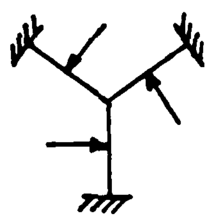

Figure 4

is characterized by a rotation about an axis. In this case, a rotation of 120° would transform the structure and its loads to an equivalent configuration.

The third type of symmetry, the center of symmetry, has as its symmetry operation an inversion through a center (e.g., Figure 5). In Figure 5, the center is labeled O.

We observe that the above specifications of the symmetry operations characterizing a particular structure's symmetry are not necessarily unique. For example, we could also characterize the symmetry of Figure 5 as a sequence of two reflections, one in the 2-3 plane followed by one in the 1-3 plane, or vice versa. Figure 5 also serves as an

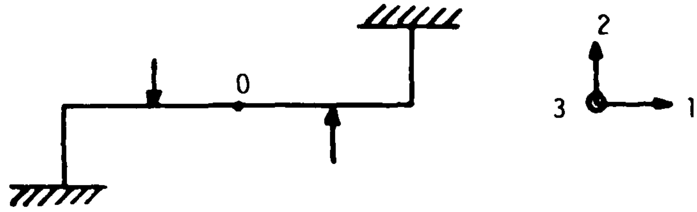

Figure 5

example of a structure with an axis of symmetry (with a rotation angle of 180°).

In general, the plane of symmetry can be viewed as the fundamental type of symmetry, since it can be shown that all symmetry transformations of finite figures in 3-D reduce to successive reflections in not more than three planes (which might not even be symmetry planes) [3].

The identification of the symmetry possessed by some structure requires not only geometrical symmetry but also symmetry with respect to material properties. For example, if the beam in Figure 1 were made of steel on the left half and aluminum on the right, there would be no symmetry to exploit. In general, many other properties may also play a role in deciding the presence of symmetry for some problems (e.g., thermal radiation problems might require symmetry with regard to color).

Finally, although loads have been included in some of the examples above, the application of symmetry, as will be seen, does not depend on the loads' being symmetric. Thus, in determining the symmetry possessed by some structure, only the structure (in the absence of loads) need be considered. In this case, the identification of the symmetry possessed by a structure is generally merely a matter of inspection.

LOADS

Once the symmetry properties of a structure are identified, the loads can be addressed. The question of whether or not a given system of loads is symmetric or not depends on the structure to which it is applied. Specifically, a system of loads, when applied to a structure possessing certain symmetry, is defined as <u>symmetric</u> if it is brought into an

equivalent configuration by the symmetry operations of the structure. The system of loads is defined as antisymmetric if the symmetry operations plus a negation of signs of all loads brings them into an equivalent configuration. For example, the loads of Figures 1, 4, and 5 are symmetric; the load system in Figure 2 is antisymmetric; the load system in Figure 3 is nonsymmetric (i.e., neither symmetric nor antisymmetric).

In general, any nonsymmetric system of loads can always be uniquely decomposed into the sum of a symmetric and an antisymmetric system of loads (e.g., Figure 6).

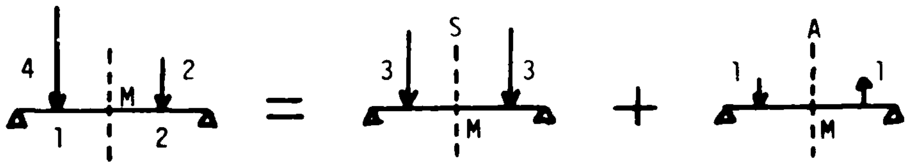

Figure 6

In Figure 6, the symmetric part of the load F_s and the antisymmetric part F_a are given by

$$F_s = \frac{1}{2}(F_1 + F_2)$$
$$F_a = \frac{1}{2}(F_1 - F_2) \qquad (4)$$

where points 1 and 2 are image points of each other.

THE GUIDING PRINCIPLE

The principle upon which all applications of symmetry are based is that "equivalent causes produce equivalent effects," or more generally, "the effect is at least as symmetric as the cause" [1,4,5]. In the context of structural mechanics, the practical effect of this principle is that symmetric loads produce symmetric effects (displacements, stresses, etc.), and antisymmetric loads produce antisymmetric effects. In any case, the structure must be symmetric in order for the principle to apply.

BOUNDARY CONDITIONS

When only a portion of a symmetric structure is modeled, the basic principle provides the tool to derive systematically the symmetry (or antisymmetry) boundary conditions.

Emphasis here will be restricted to planes of symmetry since the symmetry plane has already been identified as the fundamental type of symmetry.

Consider, for example, the beam of Figures 1 and 2. We wish to derive the symmetry and antisymmetry boundary conditions to be applied at the mid-span (point M) if only half of the beam's span were modeled. This is done by (1) considering in turn each displacement component at that point, (2) applying the symmetry (or antisymmetry) operations characterizing the structure to that component (assumed to be nonzero), and (3) observing whether or not that component may in fact be nonzero and not violate symmetry. The symmetry operation for the beam of Figure 1 is merely a reflection into the 2-3 plane containing point M. The antisymmetry operations consist of the same reflection followed by a negation of sign. For example, for the beam of Figure 1, assume $u_1 > 0$ at M. The reflection results in an image of u_1 having opposite orientation. The additional negation of sign (for antisymmetry) yields a result coinciding with the original configuration. Therefore, u_1 must vanish at M in order not to violate symmetry, but u_1 may be nonzero for nonsymmetric motion. Similarly, we find that u_2 and u_3 vanish at M for antisymmetry and may be nonzero for symmetry.

The rotational degrees of freedom require slightly different rules. Consider the following problem which is clearly symmetric. The axial vector representation of the symmetric bending moments in Figure 7 shows that the reflection of an axial vector into a plane requires an additional

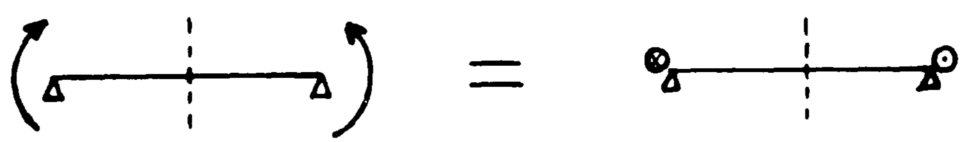

Figure 7

negation of sign compared to how ordinary vectors reflect.

Returning now to the beam of Figure 1, the application of the symmetry operation (reflection) to the rotational components R_1, R_2, R_3 indicates that, for symmetry, R_2 and R_3 must vanish in order not to violate symmetry, and $R_1 = 0$ for antisymmetry. To summarize, the boundary conditions to impose at mid-span (point M) in Figure 1 are

$$\begin{aligned} u_1 = R_2 = R_3 = 0 \quad & \text{for symmetry} \\ R_1 = u_2 = u_3 = 0 \quad & \text{for antisymmetry} \end{aligned} \tag{5}$$

Since the choice of coordinate directions in Figures 1 and 2 is arbitrary, the generalization of the results in eqs. (5) is the following: points lying in a plane of symmetry can suffer no translation out of the plane and no rotation about in-plane lines. The antisymmetry boundary conditions are that the complementary degrees of freedom are constrained. The complementary nature of the symmetric and antisymmetric boundary conditions is a general result which follows from the observation that the only distinction between antisymmetry and symmetry is an additional negation in the symmetry operations.

NONSYMMETRIC LOADS

As illustrated in Figure 6, any general loading system can always be uniquely decomposed into the sum of symmetric and antisymmetric systems. For the structure of Figure 6, for example, the analyst would model the left half, say, and solve the problem in two steps: (1) the symmetric part of the load is applied along with symmetric boundary conditions at M, and (2) the antisymmetric part of the load is applied along with antisymmetric boundary conditions imposed at M. Thus, we have Figure 8, where the symmetric (S) and antisymmetric (A) boundary

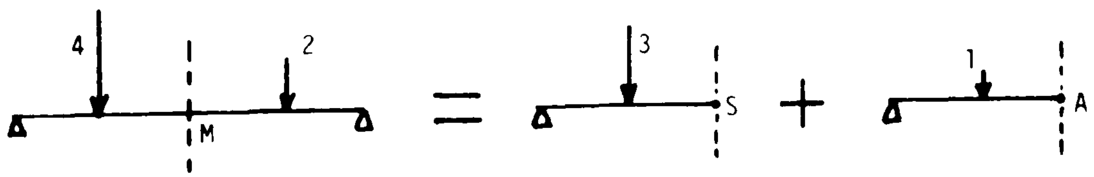

Figure 8

conditions are given in eq. (5).

Observe that adding the two solutions in Figure 8 yields the solution of the original problem only for the left side of the structure. To obtain the solution for the right side (when only the left side is modeled), the two solutions can be subtracted:

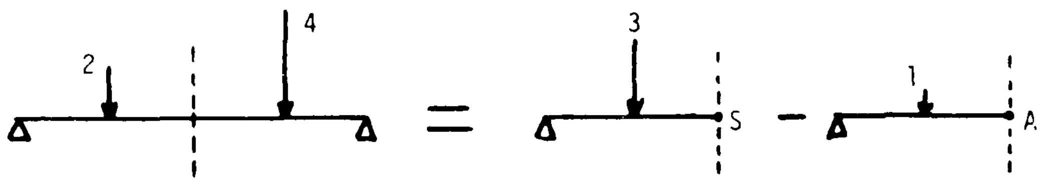

Figure 9

Taking the difference of the S and A solutions has the practical effect of reversing the role played by the left and right sides. Thus, even though only the left side is modeled, the entire solution can be obtained.

MULTIPLE PLANES OF SYMMETRY

Consider the rectangular region below possessing the two symmetry planes indicated:

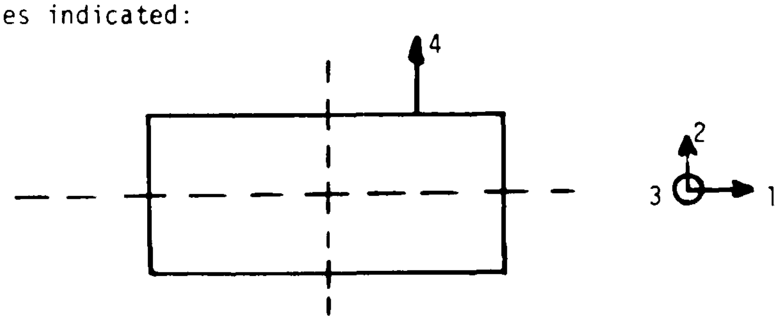

Figure 10

This problem can be decomposed into four parts, as follows:

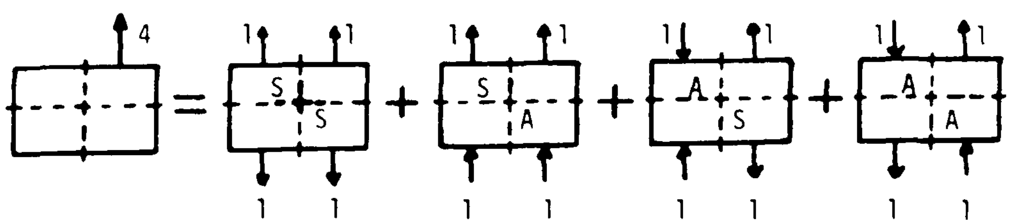

Figure 11

Here, the upper right quadrant, say, is modeled, and the four combinations of symmetry and antisymmetry boundary conditions (S-S, S-A, A-S, A-A) are imposed on the points lying in the two planes of symmetry. The four solutions can be combined in various ways to yield the solutions in all four quadrants.

FREE, UNDAMPED VIBRATIONS

The foregoing discussion has been devoted exclusively to statics problems. Free, undamped vibration problems (eigenvalue problems) can also exploit symmetry. The calculation of all natural frequencies and mode shapes would require one eigenvalue run for each unique combination of symmetry/antisymmetry boundary conditions. For example, the region of Figure 10, which has two orthogonal planes of symmetry, could be solved by modeling only one quadrant and applying, in turn, each of the four combinations of boundary conditions.

It is interesting to observe here that the total number of degrees of freedom (DOF) involved in the four component problems of Figure 10 exactly equal the original number of DOF contained in the complete problem [6]. This follows as a direct consequence of the symmetry and antisymmetry boundary conditions' involving complementary sets of DOF. Thus, we have "conservation of DOF." If this were not so, we would have the disturbing situation in which the mere application of symmetry results in the creation or destruction of DOF. The purpose of applying symmetry is, of course, to solve (with less effort) the same problem rather than a different problem.

TIME-DEPENDENT PROBLEMS

All of the preceding results for statics problems also apply to transient (time-dependent) situations, except that the entire history of time-dependent loads must be decomposed into symmetric and antisymmetric parts. This is illustrated in the context of underwater shock response in reference 7.

FINITE ELEMENTS IN SYMMETRY PLANES

Special consideration is necessary to treat the situation in which elements lie entirely in a plane of symmetry (i.e., the grid points which define the element lie entirely in the plane). For example, in the stiffened plate shown below,

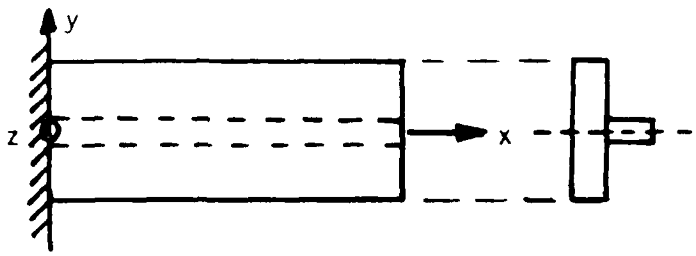

Figure 12

the beam stiffener (modeled with beam elements) lies entirely in the xz-plane, which is a plane of structural symmetry. Although the symmetry boundary conditions are unaffected by this situation, care must be exercised in computing the geometrical properties of a beam element lying in the symmetry plane. In particular, the properties for each "half-element" should be specified so that the "half-element" receives one-half the total stiffness rather than one-half the cross-section. For example, properties such as area A, cross-sectional moments of inertia I_1, I_2, and I_{12}, and torsional constant J would first be computed for the full cross-section before entering one-half of those values. (Note that J and one of I_1 or I_2 do not depend linearly on individual cross-sectional dimensions).

To prove the validity of the above approach, we need only treat each half of the symmetric structure as a "super element" involving many grid points. Then, if the two sides were to be recombined using the usual rules of matrix assembly, the resulting stiffness matrix would have to be the correct stiffness matrix for the entire structure. Thus, when an element is cut in half by a symmetry plane, each side receives one-half of the total stiffness.

A RETURN TO FIGURE 3

The third motivating example used earlier (Figure 3) is now seen as a routine application of symmetry. Let us characterize the symmetry of the structure as the sequence of two reflections, one in the 1-3 plane containing the center 0 and one in the 2-3 plane containing the center. By considering in turn each of, say, six DOF at the center point, we find that the displacement vector at point 0 must satisfy

$$\begin{aligned} u_1 = u_2 = R_1 = R_2 = 0 \quad & \text{for symmetry} \\ u_3 = R_3 = 0 \quad & \text{for antisymmetry} \end{aligned} \tag{6}$$

Thus the problem can be decomposed as follows:

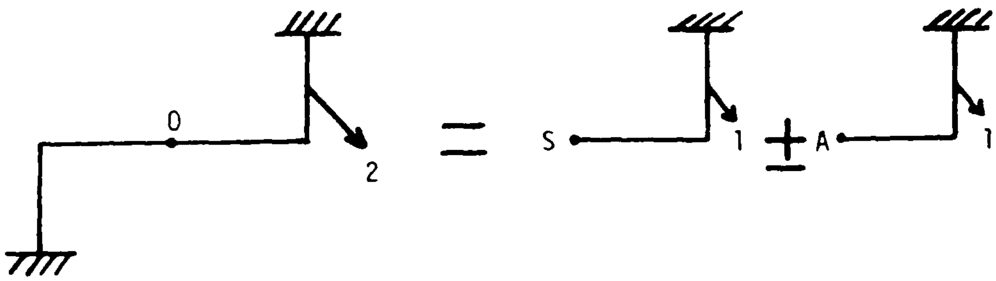

Figure 13

where the S and A boundary conditions are given in eqs. (6).

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GRID POINT SEQUENCING CONSIDERATIONS IN FINITE ELEMENT ANALYSIS

Gordon C. Everstine

COMPUTATION, MATHEMATICS, AND LOGISTICS DEPARTMENT
TECHNICAL MEMORANDUM

JULY 1977 TM-184-77-04

GRID POINT SEQUENCING CONSIDERATIONS
IN FINITE ELEMENT ANALYSIS

Summary. A brief review of the definitions of matrix bandwidth, profile, and wavefront, and their implications in finite element analysis, is presented.

BACKGROUND

Many problems of scientific and engineering interest reduce to the numerical problem of having to solve a large set of linear algebraic equations such as (in matrix form)

$$Ax = b \tag{1}$$

where the vector b and the square matrix A are known, and the unknown vector x is sought. In finite element and other applications, A is also sparsely populated (i.e., it contains far more zeros than nonzeros), since the procedure under which finite element matrices are assembled dictates that the off-diagonal matrix terms coupling any two degrees of freedom to each other are zero unless those degrees of freedom are common to the same finite element. It also follows that the locations of the nonzero matrix elements of the matrix A depend solely on the ordering of the unknowns. Thus, it is possible with sparse matrices to choose an ordering which results in the nonzeros' being located in such a way as to be convenient for subsequent matrix operations such as equation solving or eigenvalue extraction. Many such algorithms have been expressly written to operate very efficiently on matrices possessing small bandwidth, profile, or wavefront. This is accomplished by avoiding arithmetic operations on matrix elements known to be zero. As a result, the execution time for a "band solver", for example, would be $O(NB^2)$ for large N and B, where N is the matrix order and B the bandwidth. For a given structural model, N is fixed, but B depends on the ordering of the unknowns (grid points). Clearly, in this case, it is desirable to reduce B as much as possible.

DEFINITIONS

Although the definitions to be given here are reasonably standard (at least in finite element circles), uniformity of definitions and notation among the various workers in the field does not yet exist.

Given a symmetric square matrix A of order N, we define a "row bandwidth" b_i for row i to be the number of columns from the first nonzero in the row to the diagonal, inclusive. Numerically, b_i exceeds by unity the difference between i and the column index of the first nonzero entry of row i of A. Then the matrix <u>bandwidth</u> B and <u>profile</u> P are defined as

$$B = \max_{i \le N} b_i \tag{2}$$

$$P = \sum_{i=1}^{N} b_i \tag{3}$$

Let c_i denote the number of <u>active columns</u> in row i. By definition, a column j is active in row i if $j \ge i$ and there is a nonzero entry in that column in any row with index $k \le i$. The matrix <u>wavefront</u> W is then defined as

$$W = \max_{i \le N} c_i \tag{4}$$

Sometimes c_i is referred to as the <u>row wavefront</u> for row i. Since the matrix A is symmetric,

$$P = \sum_{i=1}^{N} b_i = \sum_{i=1}^{N} c_i \tag{5}$$

The wavefront W is sometimes called the <u>maximum wavefront</u> W_{max} to distinguish it from the <u>average wavefront</u> W_{avg} and <u>root-mean-square wavefront</u> W_{rms} defined as

$$W_{avg} = \frac{1}{N} \sum_{i=1}^{N} c_i = \frac{P}{N} \tag{6}$$

$$W_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} c_i^2} \tag{7}$$

As a consequence of the above definitions, it follows that, for a given matrix,

$$W_{avg} \le W_{rms} \le W_{max} \le B \le N \tag{8}$$

The first two inequalities would be equalities only for uninteresting special cases such as diagonal matrices.

We define the degree d_i of node i to be the number of other nodes to which it is connected; i.e., to be more precise, d_i is the number of nonzero off-diagonal terms in row i of the matrix A. (This implies, for example, that diagonally opposite nodes in a quadrilateral element are "connected" to each other.) Hence, the maximum nodal degree M is

$$M = \max_{i \le N} d_i \tag{9}$$

The number of unique edges E is defined to be the number of nonzero off-diagonal terms above the diagonal. Hence, for a symmetric matrix,

$$E = \frac{1}{2} \sum_{i=1}^{N} d_i \tag{10}$$

Thus the total number of nonzeros in A is 2E+N, and the density ρ of the matrix A is

$$\rho = \frac{2E + N}{N^2} \tag{11}$$

Note that, in the above definitions, the diagonal entries of the matrix A are included in b_i and c_i (and hence in B, P, W_{max}, W_{avg}, and W_{rms}). Therefore, these definitions differ from those of some authors, but conform to those, for example, in the NASTRAN literature. Except for the rms wavefront W_{rms}, it is easy to convert the various parameters from one convention (including the diagonal) to the other (not including the diagonal).

Also note that, in this context, the order N of the matrix A is sometimes taken to be the same as the number of nodes. In general finite element usage, however, each node (grid point) has several degrees of freedom (DOF), not just one. For structures having, say, six DOF per node, the actual DOF values of B, W_{max}, W_{avg}, or W_{rms} would be (in the absence of constraints) six times their corresponding grid point values.

Example

The above definitions (2)-(7) can be illustrated by the following simple example. In Figure 1 is shown a matrix of order six. In each row and column a line is drawn from the first nonzero to the diagonal. Thus b_i is the number of columns traversed by the solid line in row i.

b_i	c_i	c_i^2
1	3	9
1	5	25
3	4	16
3	3	9
4	2	4
6	1	1
Σ = 18	Σ =18	Σ = 64

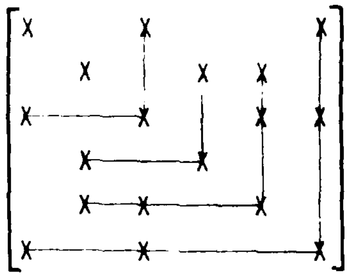


Figure 1 - Example illustrating definitions of matrix bandwidth, profile, and wavefront.

Similarly, the number of active columns c_i in row i is the number of vertical lines in row i to the right of and including the diagonal. Thus, from the above definitions, B=6, W_{max}=5, P=18, W_{avg}=3.0, and W_{rms}=3.3.

THE RELATIONSHIP TO STRUCTURES

Consider the one-dimensional six DOF system of six scalar springs shown below.

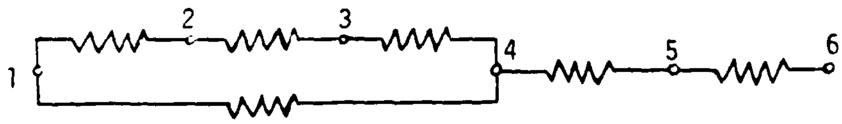


Figure 2

For each spring, the element stiffness matrix is

$$k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{12}$$

where k is the spring stiffness. For the nodal numbering indicated in

Figure 2, the 6x6 system stiffness matrix looks like

$$
\underline{K} = \begin{bmatrix} X & X & & X & & \\ X & X & X & & & \\ & X & X & X & & \\ X & & X & X & X & \\ & & & X & X & X \\ & & & & X & X \end{bmatrix} \qquad (13)
$$

where an X indicates the location of a nonzero entry. From equation (13) and the definition (2), the matrix bandwidth B is 4. Observe that the bandwidth can also be obtained directly from the structure (Figure 2) by adding unity to the maximum numerical difference between connected node numbers (node 1 is connected to node 4). The same result is also obtained for the following numbering:

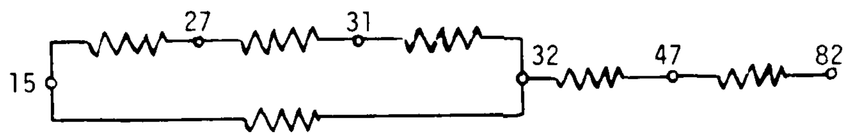


Figure 3

This is because, from the point of view of the matrix connectivity, there is no difference between the structures in Figures 2 and 3, since the ordering of the unknowns is the same. Some structural programs (e.g., NASTRAN) allow the user to specify grid point numbers as in Figure 3, rather than consecutively from 1 to N. However, in order to compute the matrix bandwidth directly from this structure (by looking at the maximum numerical difference between connected node numbers), Figure 3 would first have to be simplified to Figure 2.

To illustrate the difference that sequencing makes, consider instead the following numbering

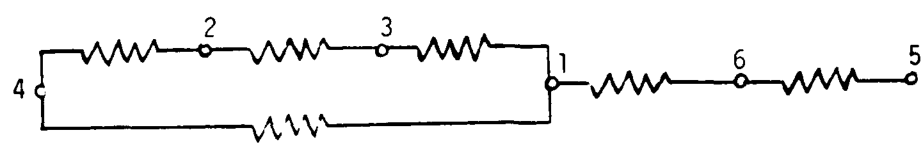


Figure 4

Here the bandwidth is 6, so that the ordering of Figure 2 is to be preferred over that of Figure 4. However, a still better sequence (i.e., one with a smaller bandwidth) is

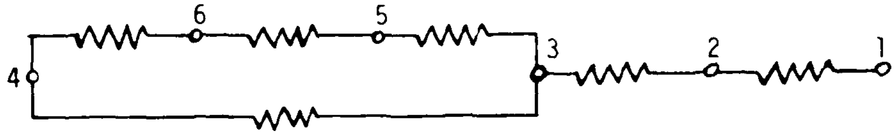
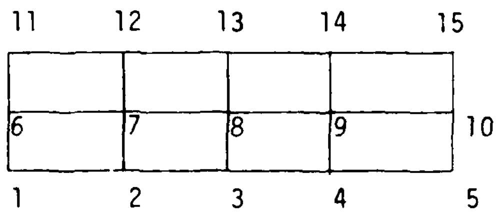


Figure 5

where B=3.

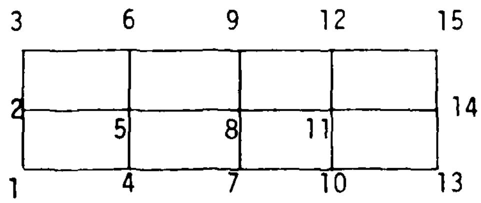
The same concepts can also be applied to two- and three-dimensional structures. For example, consider the plate below modeled with a 2x4 array of quadrilateral elements.



B=7

Figure 6

Here the grid point bandwidth is 7 (recall that all nodes in a given element are "connected" to all other nodes in the same element). A better sequence (i.e., one with a smaller bandwidth) would number first across the "short" direction ("short" in the sense of number of nodes rather than actual distance):



B=5

Figure 7

With this sequence the grid point bandwidth is now 5.

In general, the plates of Figures 6 or 7 have more than one DOF per node. Thus, although it suffices to consider only grid point bandwidth

when picking an ordering, the actual DOF bandwidth which the equation solver encounters would be much larger. For example, structures having 6 DOF/node and a grid point bandwidth of B would have a DOF bandwidth of 6B.

Although the above discussion was written from the point of view of matrix bandwidth, similar comments could be made instead from the point of view of matrix profile or wavefront. For example, NASTRAN's level 16 contains a decomposition routine which operates fastest on those matrices having smallest rms wavefront.

AUTOMATIC RESEQUENCERS

Although the preceding sections define the various terms and show how one might compute the bandwidth, profile, or wavefront for a given matrix, such calculations would clearly be very tedious for all but the smallest structures. Even more difficult, in general, is the job of resequencing the nodal labels to reduce the parameter of interest. This is especially true for large, complicated meshes or those generated automatically on a computer.

Fortunately, a large number of algorithms have been developed to automate the assignment of grid point labels, given the connectivity of the mesh. Since it is clearly impractical to check each of the N! possible sequences associated with a given matrix A of order N, each algorithm attempts some (presumably) rational strategy for arriving quickly at a grid point sequence.

For NASTRAN, for example, two preprocessors are currently being used to resequence nodes: BANDIT [1-3], which uses the Cuthill-McKee [4] and Gibbs-Poole-Stockmeyer [5] algorithms, and WAVEFRONT [6], which uses the Levy algorithm [7,8]. Some good comparisons of several resequencing algorithms have been made by Cuthill [9] and Gibbs, et al [10].

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NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Md. 20084

BANDIT USER'S GUIDE

Gordon C. Everstine

COMPUTATION, MATHEMATICS, AND LOGISTICS DEPARTMENT
TECHNICAL MEMORANDUM

MAY 1977

TM-184-77-03
Revision B (December 1978)

BANDIT User's Guide

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Numerical Mechanics Division (184)
David Taylor Naval Ship R&D Center
Bethesda, Maryland 20084

1. BANDIT version: version 9, updated [illegible] 4/78

2. Compatible NASTRAN Versions: NASA's level 17 (and below), MSC NASTRAN, Navy NASTRAN

3. Input:

   a. maximum: a standard NASTRAN data deck (ID or NASTRAN thru ENDDATA) plus $ option cards, if any

   b. minimum: $ option cards, BEGIN BULK, element connection cards, ENDDATA

4. Output:

   a. printed output

   b. punched output (SEQGP cards or entire deck)

   c. file (unit 8) containing complete deck plus SEQGP cards; this file is suitable to be used as input to NASTRAN

5. Elements Recognized:

CELAS1	CELAS2	CDAMP1	CDAMP2	CMASS1
CMASS2	CROD	CTUBE	CVISC	CDAMP3
CDAMP4	CELAS3	CELAS4	CMASS3	CMASS4
CAXIF2	CAXIF3	CAXIF4	CBAR	CCONEAX
CFLUID2	CFLUID3	CFLUID4	CHBDY	CHEXA1
CHEXA2	CHTTRI2	CIS2D4	CIS2D8	CIS3D8
CIS3D20	CONM1	CONM2	CONROD	CQDMEM
CQDMEM1	CQDMEM2	CQDPLT	CQUAD1	CQUAD2
CSHEAR	CSLOT3	CSLOT4	CTETRA	CTORDRG
CTRAPRG	CTRBSC	CTRIA1	CTRIA2	CTRIARG
CTRMEM	CTRPLT	CTWIST	CWEDGE	CDUMMY
CDUM1	CDUM2	CDUM3	CDUM4	CDUM5
CDUM6	CDUM7	CDUM8	CDUM9	CTRIAX6
CTRIM6	CDAMP4*	CELAS4*	CMASS4*	CDAMP2*
CELAS2*	CMASS2*	CONM1*	CONM2*	CONROD*
CIHEX1	CIHEX2	CIHEX3	CTRAPAX	CTRIAAX
CQUADTS	CTRIATS	CQDMEM3	CHEX8	CHEX20
CTRPLT1	CTRSHL	CRIGD1	CRIGD2	CRIGDR
CBEAM	CFTUBE	CHEXA	CPENTA	CQUAD4
CTRIA3				

6. Reduction Approach: Uses Cuthill-McKee (CM) and/or Gibbs-Poole-Stockmeyer (GPS) methods to reduce matrix bandwidth, profile, wavefront, or rms wavefront.

7. Core Requirements:

Total Core = Program + Working Storage

where Program = $47K_8$ words on CDC
= 145K bytes on IBM
= 24K words on UNIVAC, Honeywell

Working Storage Required = $(\frac{M}{NW} + 8)N$

where N = number of grid points
M = maximum nodal degree (the maximum number of nodes connected to any node)
NW = integer packing density (integers/word)
Working Storage = open core on open core versions of BANDIT

CDC: NW = 6 for $N \le 510$
5 for $510 < N \le 2045$
4 for $2045 < N \le 16380$
3 for $16380 < N \le 524286$

IBM: NW = 2 for $N \le 32766$

UNIVAC, Honeywell: NW = 4 for $N \le 508$
3 for $508 < N \le 4095$
2 for $4095 < N \le 262142$

8. $ Option Cards:

a. location: anywhere before BEGIN BULK
b. general format: $KEYWORD1 KEYWORD2
c. rules:
(1) $ in column 1
(2) KEYWORD1 starts in column 2
(3) keywords separated by one or more blanks
(4) no embedded blanks in keywords
(5) the first two letters of each keyword are required for recognition

9. $ Option Cards: (default underlined)

a. For General Use:

$SEQUENCE (NO, YES) — Is resequencing to be performed?

$GRID N — Upper bound on number of grid points.

This card must be used with solid elements and MPC's since default M (maximum nodal degree) is about 19. It is recommended to use it with all runs since it is used for efficient allocation of core.

$CONFIG N — Computer model (from NASTRAN manual).

Used in estimating NASTRAN decomposition time.

$CONFIG N,M,L — N = computer model (from NASTRAN manual)

M = computer for which decomposition time estimate is desired if different from one BANDIT is on (M=1 for CDC, 2 for IBM, 3 for UNIVAC).

L = flag to request printout of all NASTRAN multiply-add time constants (0 = no, 1 = yes)

$PUNCH (NONE, SEQGP, ALL) What should be punched?

$CRITERION (BAND, PROFILE, WAVEFRONT, RMS) What should be reduced?

Recommendations:

BAND for NASTRAN Level 15.5 and below
RMS for NASTRAN Level 15.9 and above and MSC NASTRAN

$METHOD (CM, GPS, BOTH) By what method?

$MPC (NO, YES) — Take MPC's into account?

"YES" generates, for each MPC equation in deck, additional connections between the independent points and every other point to which the dependent point is connected. Dependent points can be eliminated from connection table by using $IGNORE.

$PRINT (MIN, MAX) — What printed output?

"MIN" is adequate for most purposes. "MAX" generates additional connection tables and nodal lists.

$IGNORE G1,G2,... — Grid points to ignore.

Nodes ignored are eliminated from the connection table and sequenced last. This should be used, for example, for nodes of very high degree compared to other nodes in the structure.

$ADD N — Add N to new sequence numbers.

May be used to avoid duplication of internal numbers if not all nodes of a structure are being sequenced in one run.

$ELEMENTS (NO, YES)   List BANDIT's element library?

$APPEND CNAME NCON IFLD   User-defined connection card.

CNAME = name of connection card (e.g., CBAR) left-adjusted starting in column 9.

NCON = number of connections on card (i.e., nodes in element) ≥ 1.

IFLD = NASTRAN field number on parent card in which first connection appears ≤ 9.

NCON and IFLD may appear anywhere in columns 17-32 separated by one or more blanks. No long-field connection cards may be defined. Connections must be listed consecutively on parent and continuation cards, if any. Each $APPEND card defines a new element type.

b. For Particular Users:

$NASTRAN (NO, YES)   NASTRAN to follow BANDIT? (IBM)

"YES" generates a condition code 5 after a successful completion.

$INSERT   Location of cards to insert.

$INSERT N   Number and location of cards to insert.

May be used by remote users to insert checkpoint dictionary from disk file into executive control deck.

$LINES N   Number of printed lines per page.

$PLUS +   User-defined plus sign.

Allows user to input his own special plus sign, if necessary.

$DIMENSION N   Dimension of a scratch array.

$HICORE N   Amount of core requested in words. (UNIVAC)

c. For Program Developer :

$TABLE (NO, YES)   Output connection table?

$START G1,G2,...   User-supplied CM starting nodes.

$DEGREE N   Ignore nodes of degree exceeding N.

$SPRING (NO, YES)   Generate scalar springs?

The springs (CELAS3) have same connectivity as original structure.

10. Installation-dependent Remarks:

a. On CDC machines, the automatic reduction of field length at execution time should be suppressed, e.g., with an RFL card.

b. Unless modified locally, IBM and Honeywell versions are not open core programs, but fixed core. Hence, calls by BANDIT for more core require the change of two statements in the main program.

11. Additions to BANDIT version 9:

a. Open core and HICORE on UNIVAC.

b. Eliminate backspace of Unit 5 on IBM and Honeywell.

c. Min. nodal degree printout in summary.

d. User-selected CM starting nodes fix.

e. Case Control card counter.

f. New Level 17 configurations and time constants.

g. Subroutine READIT efficiencies.

h. Recovery of SEQGP cards generated by CM if abort in GPS due to exceeding scratch dimension.

i. $APPEND card to define connection card at execution time.

j. CRIGD1 with THRU option.

k. Warning for illegal ENDDATA format.

l. Optional printout of multiply-add time constants.

m. Reset of $DIMENSION value if $GRID declared.

n. Time and disk space efficiencies with MPC equations.

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